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On the theory of porous elastic rods

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ABSTRACT

We consider the direct approach to the theory of rods, in which the thin body is modelled as a deformable curve with a triad of rigidly rotating orthonormal vectors attached to every material point. In this context, we employ the theory of elastic materials with voids to describe the mechanical behavior of porous rods. First, we derive the dynamical nonlinear field equations of the model. Then, in the framework of linear theory, we prove the uniqueness of the solution to the associated boundary-initial-value problem. We identify the relevant field quantities from the theory of directed curves by comparison with the three-dimensional equations of straight porous rods. Finally, for orthotropic and homogeneous rods, we determine the constitutive coefficients in terms of the three-dimensional elasticity constants by solving several problems in the two different approaches.

1. Introduction

The theory of rods is one of the oldest fields of mechanics. The first significant studies on the behavior of thin rods have been elaborated in the 17th century by Galilei and Bernoulli, then in the 18th century by Euler and D'Alembert, and in the 19th century by Clebsch and Kirchhoff, among others. Due to these contributions in the theory of rods, several new mathematical concepts and mechanical challenges have been formulated, which played an important role in the history of development of natural sciences.

Nowadays, the modern studies on the mechanical behavior of beams and rods have received considerable attention. The growing interest in this field is due to the intensive use of rod-like structures in mechanical and civil engineering. The emergence of new technologies and advanced materials in connection with rod manufacturing lead to the necessity of elaborating adequate models and to extend the existing theories.

In general, the theories of beams and rods allow for the approximate analysis of the stress–strain state of three-dimensional bodies for which two dimensions are much smaller in comparison with the third one. In this sense, all these theories are based on the thinness hypothesis. To obtain a set of one-dimensional approximate equations, one of the following main directions can be pursued: the application of kinematical and/or stress hypotheses, the use of mathematical techniques like series expansions and asymptotic analysis, and the so-called *direct approach* based on the deformable curve model. As examples in the first direction we can mention the beam theories of Euler and of Timoshenko, see e.g. Timoshenko (1921), Svetlitsky (2000), Hodges (2006) for an extensive account. Mathematical techniques for the study of rods and beams include the use of formal asymptotic expansions (Trabucho and Viaño, 1996; Yu et al., 2002; Tiba and Vodak, 2005; Berdichevsky, 2009), the Γ-convergence analysis (Freddi et al., 2007) and other variational methods (Meunier, 2008; Sprekels and Tiba, 2009).

The direct approach has been introduced for the first time by the Cosserat brothers (Cosserat and Cosserat, 1909). As a model for rods, they have considered a deformable curve in which every material point is connected to a triad of orthonormal vectors (also called *directors*) to characterize its orientation. Later, this idea has been modified and developed by Green and Naghdi (Green et al., 1974; Green and Naghdi, 1979) who created the so-called theory of Cosserat curves, in which every material point is attached to a pair of deformable directors. We mention that the Cosserat theory for rods was developed in parallel with the Cosserat theory for shells. These two models have been presented and analyzed in details in the books of Antman (1995) and Rubin (2000).

Another direct approach for shells and rods has been elaborated by Zhilin (1976, 2006a,b, 2007), who followed the original idea of the Cosserats and considered deformable continua (surfaces or curves) endowed with a triad of rigidly rotating orthonormal vectors connected to each point. In his approach, Zhilin has supplemented the kinematical model suggested by Cosserat with appropriate constitutive equations, thus making the model applicable to solve practical problems (Altenbach et al., 2006). We also mention that the latter approach has the attribute of simplicity, in comparison with the theories of Cosserat surfaces or curves given by Green and Naghdi. The approach to rods proposed by Zhilin is also called the theory of *directed curves*.

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