



Finite strain hyperelastoplastic modelling of saturated porous media with compressible constituents

A. Gajo*

Dipartimento di Ingegneria Meccanica e Strutturale, Università di Trento, Via Mesiano 77, 38050 Povo, Trento, Italy

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ABSTRACT

Starting from the general expression of the free-energy function of a saturated porous medium at finite deformations in the case of compressible fluid and solid constituents, and from the internal dissipation increment, the general expressions of the plastic potential and flow rule are deduced together with the general form of the consistency condition. Reference is made to an elementary volume moving with the solid skeleton in a Lagrangian description, which is treated as an open system from which the pore fluid can flow freely in and out.

As a result, a *generalisation* is provided of the classical Prandtl–Reuss relationship of small strain elastoplasticity in single-phase media to finite strain *multiplicative* (for \mathbf{F}) and *additive* (for the fluid mass content) elastoplasticity in saturated porous media with compressible constituents.

The following particular cases are analysed in detail: null plastic volume change of the solid constituent, incompressibility of the solid constituent, incompressibility of both fluid and solid constituents, quasi-linear theory (in which the solid constituent is assumed to be nearly incompressible, and therefore undergoing small volume changes), and geometrically linearised theory. The simplified approaches previously presented in the literature are thus recovered within a unified framework and new, simplified constitutive assumptions are made.

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1. Introduction

Papers dealing with the elastoplastic behaviour of porous media at finite strains typically reflect the two historical approaches in the literature to the constitutive modelling of porous media, namely: mixture theories and the so-called ‘purely macroscale theories’. In the former, the porous media are represented by superposed and interacting continua and the field equations of each constituent are derived from averaging processes (Morland, 1972; Bowen, 1976, 1982), whereas the latter were first proposed by Biot (1972, 1973, 1977) and later by Coussy (1989), and assume that the standard concepts of continuum mechanics are still relevant on a macroscale (Coussy et al., 1998). It is worth recalling that it is generally accepted that an equation is missing in mixture theory for a saturated porous medium (e.g. Svendsen and Hutter, 1995), so several approaches have been suggested to overcome this drawback (see De Boer, 1996, for a review of the various proposals). Within this framework, Gray and Müller (2005) have recently proposed a thermodynamically constrained averaging theory starting from the microscale constituent continua to the macroscale.

This approach was later applied by Gray and Schrefler (2007) to small-strain, multiphase media, recovering the traditional form of Biot’s coefficients.

Within the framework of mixture theories, the first papers taking the elastoplastic behaviour of the solid material into account are those by Morland (1972), Kojić and Cheatham (1974), De Boer and Kowalski (1983), and De Boer and Ehlers (1986), each of them based on different simplifying assumptions, such as geometrically linearised theory, the special multiplicative decomposition of the finite deformation gradient \mathbf{F} , the incompressibility of the fluid and solid constituents, or the small, elastic strains of the solid constituent. Among the first works based on ‘purely macroscale theories’, there are those by Carter et al. (1977, 1979) and Prevost (1980), that consider incompressible solid constituents and are based on the Jaumann stress rate in an updated Lagrangian approach (see also Meroi et al., 1995). In particular, the latter assumption may lead to unphysical responses (Johnson and Bammann, 1984), and that is why the most recent solutions are based on hyperelastic formulations.

It is generally agreed that the assumption of incompressibility of the solid grains *immensely* simplifies the relationships (Bennethum, 2006), because the *coupling* between the solid and fluid phases is much weaker in this case. That is why the compressibility of the solid constituent has mostly been neglected in the

* Tel.: +39 0461 882519; fax: +39 0461 882599.

E-mail address: alessandro.gajo@ing.unitn.it

URL: <http://www.ing.unitn.it/~gajoal/>