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# Physical variational principle and thin plate theory in electro-magneto-elastic analysis

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Under the assumption of the quasi-static electric and magnetic fields the electro-magneto-elastic analysis including medium and its environment is studied in this paper. The complete governing equations under the finite deformation can be derived from the physical variational principle. In the physical variational principle the variations of the electric potential and magnetic potential are divided into local variations and migratory variations. From the virtual change of the sum of the electromagnetic energy and the couple energy produced by the migratory variation we can get the electromagnetic force and in this case the virtual variation of the volume should be considered. It is also found that the Maxwell stress is directly related to the strain in a material with piezoelectric or piezomagnetic behavior for the finite deformation case. The thin plate theory in first order is derived from the general theory in this paper and the Maxwell stress is naturally included in the governing equations.



## 1. Introduction

The electro-magneto-elastic analyses in elastic electromagnetic media under the quasi-static electric and magnetic fields are very important due to its extensive applications in the engineering structures. In the previous literatures various theories were presented. Landau and Lifshitz (1960) and Stratton (1941) gave the derivation of the Maxwell stress in electroelastic media from the energy principle, but their formulas were not very clear and did not give an entire variational principle. Moon (1984) systematically discussed the magnetoelastic problem. Many important fundamental theories and formulas for the electro-magneto-elastic dynamics in finite deformation were established by Maugin (1988), Eringen and Maugin (1989), Pao (1978) and others. Some differences between these theories, especially in the electromagnetic force, were discussed in Kuang (2008a), Kuang (2008b), Kuang (2009a) Jiang and Kuang (2004), Jiang and Kuang (2006), Zhou and Zheng (1997) and others. Some variational principles of electroelastic and magnetoelastic materials were discussed by Toupin (1956), Bustamante et al. (2008), etc. In our papers (2008a,2008b,2009a,2009b) we showed that together with the first law of thermodynamics, the known facts show that the following physical variational principle is also held. When variations of variables in the following variational formula are independent each other we have

$$\int_V \delta\Phi\,\mathrm{d}V = \delta W + \delta Q, \quad \text{or} \quad \int_V \delta\Phi\,\mathrm{d}V - \delta W - \delta Q = 0 \tag{1}$$

where $\delta$ is the variation sign, $\Phi$ is the internal energy per volume, $W$ is the work done by the external force and electromagnetic field, $Q$ is the heat supplied by the external heat source. In general $W$ and $Q$ are not the state functions, so they are expressed in differential forms under volume integrals. In this paper we only discuss the isothermal reversible case with $\delta Q = 0,\ \delta T = 0$. The physical variational principle with using the electromagnetic Gibbs free energy may be the most important one which will be discussed in this paper. Let $g$ be the electromagnetic Gibbs free energy per volume:

$$\delta g - \delta w^* - \delta Q = 0, \quad g = \Phi - Ts - E_i D_i - B_i H_i \tag{2}$$

where $w^*$ is the sum of the work of the external force on the medium and the complementary work of the medium on the electromagnetic field. $T$ and $s$ are the temperature and the entropy per volume respectively, $\mathbf{E},\ \mathbf{D},\ \mathbf{H},\ \mathbf{B}$ are the electric field strength, electric displacement, magnetic field strength and magnetic induction density respectively. Under the quasi-static or static electric and magnetic fields we have

$$\mathbf{E} = -\nabla\varphi, \quad \mathbf{H} = -\nabla\psi \tag{3}$$

where $\varphi$ is the electric potential and $\psi$ is the magnetic scalar potential.

In this paper we discuss the physical variational principle and Maxwell stress in electro-magneto-elastic analysis under finite deformation. In our previous papers (2008b,2009a) we assumed that the finite strain is still not too large, so we can neglect the influence of strains on the Maxwell stress. But when the finite strains cannot be neglected, we shall show that the Maxwell stress is related to strains when the initial configuration is taken as a reference configuration.

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