



# Full-state feedback controller design with “delay scheduling” for cart-and-pendulum dynamics

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## ABSTRACT

A new approach is proposed to design fixed full-state feedback controllers for linear-time-invariant (LTI) systems with multiple time-delays. This approach takes advantage of the recently introduced “delay scheduling” concept, which opens a new direction in synthesizing the control. “Delay scheduling” strategy suggests further prolonging the existing (and unavoidable) delays in the feedback in order to recover stability or to improve the control performance features. To be able to do this, however, system should have large (and maybe multiple) stable operating zones in the domain of the delays. The main contribution of this paper is to develop a methodology for designing a control law to create this feature. The operation starts with a selection of the feedback gains for a stable non-delayed system. We then utilize a recent paradigm, Cluster Treatment of Characteristic Roots (CTCR), to examine the stability outlook when the delays are present in the dynamics. A scheme is also introduced to modify this gain structure so that the system exhibits a desirably large stable pocket(s) to enable the “delay scheduling”. The paper describes the methodology, without loss of generality, on a fully-actuated cart–pendulum system. Relevant experiments are carried out to show the viability of the proposed idea.

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## 1. Introduction and problem statement

“Delay scheduling” is a recently introduced concept for time-delay system stabilization [8,9]. Assuming that a system has some unavoidable delays in the feedback line, which result in unstable dynamics, the answer to the following question is very critical: Is it possible to find some larger delay compositions that may recover the stability? The controller can obviously prolong the present delays, simply by introducing additional “hold buffers” in the feedback line. These extended delays may re-establish the stability; they may even introduce more attractive performance features. This concept of selectively increasing the delays is called the “*delay scheduling*” from which some counterintuitive results arise, as explained later in the text.

In order to deploy “delay scheduling” in a systematic manner, one needs to know the stability outlook of the system in the space of the delays. If we can declare the stable operating regions exhaustively for all possible delay compositions, this would describe the stability robustness of the system against uncertain but fixed time-delays as long as they remain within the stability regions. We can further argue that the delays can be time varying, and robustness would still hold, so long as the rate of change in delays is slow vis-à-vis the settling times of the system. We call this

problem “the stability robustness against uncertain delays”. Without such a stability picture, it is not possible to perform the “delay scheduling”. For this we use a recent technique, called the Cluster Treatment of Characteristic Roots (CTCR) [1–6]. This technique reveals the complete set of delay compositions exhaustively where the stability is guaranteed.

The main idea behind the “delay scheduling” lies in the knowledge of the stability regions (pockets) in the domain of the delays. If a particular delay composition falls in the unstable operating region, we search for stable regions with larger delays (as the controller can increase but cannot reduce the existing delays). If there is only one single stable region and it does not contain the existing delay(s), can one redesign the controller to enlarge this region? Can one, then, increase the present delay(s) and improve the control performance? A practical strategy is needed to create such a controller and we propose one in this paper. This is the main contribution here, and it is experimentally verified using a fully-actuated cart-and-pendulum system (shown in Fig. 1), without loss of generality of the treatment.

The control objective for the experimental work is to make the two-degree-of-freedom cart–pendulum system track two respective desired trajectories. Two independent delays are considered in the two feedback lines. We propose a control synthesis methodology for this, which results in an enlarged stable region in the delay space. With the new control logic in place we schedule the delays for a stable operation.

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