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A generalization of prime ideal

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Abstract: In this paper, we introduce the notion of pseudo-prime submodules of modules as a generalization of the prime ideal of commutative rings.
Keywords: Pseudo-prime submodule, pseudo-primeful, pseudo-injective, Topological module.

1 Introduction

In this paper we introduce a generalization of prime ideals of a ring. We use it to define new classes of modules. We investigate some algebraic properties of these new classes.

Throughout the paper, all rings are commutative with identity and all modules are unital. For a submodule N of an R-module M, $(N :_R M)$ denotes the ideal $\{r \in R \mid rM \subseteq N\}$ and *annihilator* of M, denoted by $\mathrm{Ann}_R(M)$, is the ideal $(\mathbf{0} :_R M)$. If there is no ambiguity we will write $(N : M)$ (resp. $\mathrm{Ann}(M)$) instead of $(N :_R M)$ (resp. $\mathrm{Ann}_R(M)$).

2 Pseudo-Prime Submodules

Definition 2.1. *Let M be an R-module.*

1. *A proper submodule N of M is called* pseudo-prime *if $(N :_R M)$ is a prime ideal of R.*

2. *We define the* pseudo-prime spectrum *of M to be the set of all pseudo-prime submodules of M and denote it by X^R_M. If there is no ambiguity we write only X_M instead of X^R_M. For any prime ideal $I \in X_R = \mathrm{Spec}(R)$, the collection of all pseudo-prime submodules N of M with $(N : M) = I$ is designated by $X_{M,I}$.*

3. *For a submodule N of M we define $V^M(N) = \{L \in X_M \mid L \supseteq N\}$. If there is no ambiguity we write $V(N)$ instead of $V^M(N)$.*

4. *When $X_M \neq \emptyset$, the map $\psi : X_M \to \mathrm{Spec}(R/\mathrm{Ann}(M))$ defined by $\psi(L) = (L : M)/\mathrm{Ann}(M)$ for every $L \in X_M$, will be called the* natural map *of X_M. An R-module M is called* pseudo-primeful *if either $M = (\mathbf{0})$ or $M \neq (\mathbf{0})$ and the natural map of X_M is surjective.*

5. *M is called* pseudo-injective *if the natural map of X_M is injective.*

Remark 2.2.

1. *By our definition, the prime ideals of the ring R and pseudo-prime submodules of the R-module R are the same. This shows that pseudo-prime submodule is a generalization of the notion of prime ideal to the modules.*

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