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Portfolio optimization problem with default risk

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Abstract

In this paper, we consider a stochastic portfolio optimization problem with default risk on an infinite time horizon. An investor dynamically chooses a consumption rate and allocates the wealth into the securities: a perpetual defaultable bond, a money market account with the constant return and a default-free risky asset. The goal is to choose the optimal investment to maximize the infinite horizon expected discounted power utility of the consumption policies (controls). The default risk premium and the default intensity are assumed to rely on a stochastic factor formulated by a diffusion process. We study the optimal allocation and consumption policies to maximize the infinite horizon expected discounted non-log HARA utility of the consumption, and we use the dynamic programming principle to derive the Hamilton–Jacobi–Bellman (HJB) equation. Then we explore the HJB equation by employing a so-called sub–super solution approach. The optimal allocation and consumption policies are obtained in terms of the classical solution to a PDE. Finally, we get an explicit formula for the optimal control strategy. In this article The soloutions are then used in portfolio management subject to default risk and derive the optimal investment and consumption policies.
Keywords: Portfolio optimization, Default risk, HJB equation, consumption policies,

1. Introduction

Merton proposed the strategy that maximizing the total expected discounted utility of the consumption for a market investment problem. Fleming and Pang discussed a classical Merton portfolio optimization problem, where the interest rate r was assumed to be an ergodic Markov diffusion process.Bieelecki and Jang studied an optimal allocation problem associated with a defaultable risky asset and there the goal was to maximize the expected HARA utility of the terminal wealth. Hou and Jin employed an intensity-based approach for the defaultable market and assumed that each investor receives a proportion of the market value of the debt prior to the default if a default occurs. Jang suggested a dynamics for the price of a defaultable bond, and studied the expected discounted utility of the wealth when the default risk premium and intensity were assumed to be constants. In this article, we investigate a portfolio optimization problem with default risk,and suggested a dynamics for the price of a defaultable bond, and studied the expected discounted utility of the wealth when the default risk premium and intensity were assumed to be constants. In this article, we investigate a portfolio optimization problem with default risk. An investor dynamically chooses a consumption rate and allocates the wealth into the securities: a perpetual defaultable bond, a money market account with the constant return and a default-free risky asset. Here the goal is to maximize the infinite horizon expected discounted utility of the consumption.There, the post-default HJB equation admitted a constant solution and the pre-default HJB equation is a linear uniformly elliptic equation with variable coefficients. For the non-log utility case, we find that the HJB equation is nonlinear. Due to its