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# European Option Pricing with Transaction Costs

**Somayeh Pourghanbar*** **Mojtaba Ranjbar†**
**Azarbaijan Shahid Madani University*
*E-mail:* s.pourghanbar@azaruniv.edu

*†Azarbaijan Shahid Madani University*
*E-mails:* m.ranjbar@azaruniv.edu

**Abstract**
This paper deals with the construction of a finite difference scheme for a nonlinear Black-Scholes partial differential equation modelling stock option pricing in the realistic case when transaction costs arising in the hedging of portfolios are taken into account. The analysed model is the Barles-Soner one.

**Keywords and phrases:** Nonlinear Black-Scholes equation, European option, Transaction costs

## 1. Introduction

It is well known that Black-Scholes model is acceptable in financial markets where one assumes that volatility is observable or transaction costs are not taken into account. Under the transaction costs, the continuous trading required by the hedging portfolio is prohibitively expensive, [1]. Several alternatives lead to pricing models that are equal to Black-Scholes one but with an adjusted volatility denoted by $\sigma$,

$$V_t + \frac{1}{2}(\sigma(S,t,V_S,V_{SS}))^2 S^2 V_{SS} + rSV_S - rV = 0, \quad S > 0, t \in [0,T[, \tag{1.1}$$

where $V$ is the option value that is a function of the underlying security $S$ and the time $t$. Here $r \geq 0$ denotes the riskless interest rate. There are some models for volatility $\sigma$, [2, 3, 4]. A more complex model has been proposed by Barles and Soner [1], assuming that investor's preferences are characterized by an exponential utility function. In their model the nonlinear volatility reads

$$\sigma^2 = \sigma_0^2(1 + \psi[e^{r(T-t)} a^2 S^2 V_{SS}]), \tag{1.2}$$

where $T$ is the maturity, and $a = \mu\sqrt{\gamma N}$, with risk aversion factor $\gamma$ and the number $N$ of options to be sold. When $a = 0$, there is no transaction cost and classical Black-Scholes equation is recovered. The function $\psi$ is the solution of the nonlinear initial value problem

$$\psi'(A) = \frac{\psi(A) + 1}{2\sqrt{A\psi(A)} - A}, \quad A \neq 0, \quad \psi(0) = 0. \tag{1.3}$$