sinc functions with application to finance

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Abstract:

In this paper, we consider Black-Scholes equation, that arises in the American option model when the stock price follows a diffusionprocess with jump components. We use the sinc method to solve this equation with its boundary conditions andfind thenumerical solution of the generalized Black–Scholes partial differential equation.The advantage of our method is that the sinc functions satisfies the boundary conditions and vanishes in infinity. This method has simple programing and gives the goodResult compared to the previous methods. The quadrature, based on sinc functions, is very accurate and can be used to approximate the neededintegrals.

Keywords: American option, Jump diffusion process, sinc function, sinc method.

1. Introduction

In financial world, an option gives its holder the right (but not the obligation) to buy or sell a prescribed risky asset from the writer for a prescribed fixed price on or before a prescribed time in the future. The fixed prescribed price is called the exercise price or strike price, and the prescribed time in the future is called the maturity date or expiry date.
There are different types of options for various purposes, for example, vanilla options (European call or put option, American call or put option etc.).
In contrast to the original Black–Scholes framework where for most of the option valuation problems closed-form formulas exist or standard numerical methods could be applied, in nonstandard financial models more complicated and precise techniques are required. We have chosen to extend the Black–Scholes model, by exploring the jump diffusion (Poisson) model, see [6] and sinc method for approximation of the option prices.

2. Mathematical model: The modified Black–Scholes PDE

The Black-Scholes equation is a partial differential equation, which describes the price of the option over time. The key idea behind the equation is that one can perfectly hedge the option by buying and selling the underlying asset just the rights way and consequently eliminate risk. This hedge implies that there is only one right price gor the option, as returned by the Black-Scholes formula given in following:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0 \tag{1}$$

As a result the Black–Scholes equation (1) is modified as a partial integral–differential equation by adding an integral part that reflects the jump diffusion structure of the process [2]: