

A Survey on exact analytical and numerical solutions of some S.D.E.s based on martingale approach and changing variable method

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Abstract

In this paper, we decide to represent analytical and numerical solutions for stochastic differential equations, specially reputed and famous equations in pricing and investment rate models. By making martingale process from an arbitrary process in $L^2(\mathbb{R})$ space, we infer equations just with stochastic part (drift free). This method could be done by Ito product formula on initial process and an appropriate martingale process, then we compare simulating method of arising this new equation with other simulating method like as E.M. and Milstein. Another suitable method is converting S.D.E.s to O.D.E.s whom we try to omit diffusion part of stochastic equation. Afterwards, it could be solved by different numerical methods like as Runge-kutta from fourth order. In this paper, we solve well known equations such as Gampertz diffusion and logistic diffusion by this method. Another powerful one is change of variable method whom we could analysis and survey a well known group of stochastic equations like as special case of squared radial Langevin process, Cox-Ingersoll-Ross model and Ornstein-Uhlenbeck process. For numerical solution of these stochastic equations, we could apply wiener chaos expansion method whom we have described in other paper.

Keywords and phrases: Numerical solution, Ito formula, Stochastic equations, Wiener chaos expansion.

1. INTRODUCTION

As we know, analytical solution of partial and ordinary differential equations has been custom since long time ago. This kind of solutions are so important especially in physics and engineering [2, 4, 6, 9]. But most of