American Options Pricing by Using Stochastic Optimal Control Problems

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Abstract
Stochastic optimal control problems frequently occur in Economics and Finance. Dynamic programming method represents the most known method for solving optimal control problems analytically. As analytical solutions for problems of optimal control are not always available, finding an approximate solution is at least the most logical way to solve them. In this paper, we present some of the basic ideas which are in current use for the solution of the dynamic programming equations. Also, based on the Markov chain approximation techniques, a numerical procedure is constructed for solution of stochastic optimal control problems. We focus on the approximation in value space method. And the Jacobi and Gauss-Seidel relaxation (iterative) methods are discussed. These are fundamental iterative methods which are used in value space approach. Finally, American options pricing are presented as simplest control problem which is called optimal stopping problem.

1. Introduction

Optimal control's models play a prominent role in a range of application areas, including aerospace, chemical engineering, robotic, economics and finance. It deals with the problem of finding a control law for a given system such that a certain optimality criterion is achieved. A controlled process is the solution to an ordinary differential equation which some parameters of the ordinary differential equation can be chosen. Hence, the trajectory of the solution is obtained. Each trajectory has an associated cost, and the optimal control problem is to minimize this cost over all choices of the control parameter. Stochastic optimal control is the stochastic extension of this; In fact, a stochastic differential equation with a control parameter is given. Each choice of the control parameter yields a different stochastic process as a solution to the stochastic differential equation. Each path wise trajectory of this stochastic process has an associated cost, and we seek to minimize the expected cost over all choices of the control parameter. In describing a stochastic control model, the kind