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Statistical quasi-Cauchy sequences

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ABSTRACT

A subset E of a metric space (X, d) is totally bounded if and only if any sequence of points in E has a Cauchy subsequence. We call a sequence (x_n) statistically quasi-Cauchy if $st - \lim_{n\to\infty} d(x_{n+1}, x_n) = 0$, and lacunary statistically quasi-Cauchy if $S_\theta - \lim_{n\to\infty} d(x_{n+1}, x_n) = 0$. We prove that a subset E of a metric space is totally bounded if and only if any sequence of points in E has a subsequence which is any type of the following: statistically quasi-Cauchy, lacunary statistically quasi-Cauchy, quasi-Cauchy, and slowly oscillating. It turns out that a function defined on a connected subset E of a metric space is uniformly continuous if and only if it preserves either quasi-Cauchy sequences or slowly oscillating sequences of points in E.

1. Introduction

The concept of a metric plays a very important role not only in functional analysis and topology, but also in other branches of sciences involving mathematics, especially in computer sciences, information theory, biological sciences, and dynamical systems.

A subset E of a metric space (X, d) is totally bounded if it has a finite ε-net for each $\varepsilon > 0$, where a subset A of E is called an ε-net in E if $E = \bigsqcup_{a\in A}[E \bigcap B(a, \varepsilon)]$. This is equivalent to the statement that any sequence of points in E has a Cauchy subsequence. This suggests that we ask what happens if we replace the term "Cauchy" with another term, "quasi-Cauchy". In fact, we could interchangeably put any of the terms "quasi-Cauchy", "statistically quasi-Cauchy", "lacunary statistically quasi-Cauchy", and "slowly oscillating", instead of the term "Cauchy".

The purpose of this paper is to investigate characterizations of the total boundedness of a subset of a metric space X, and characterizations of the uniform continuity of a function defined on a connected subset of X via the sequences mentioned above.

2. Preliminaries

Throughout this paper, **N**, R, and X will denote the set of positive integers, the set of real numbers, and a metric space with a metric d, respectively. We will use boldface letters $\mathbf{x}, \mathbf{y}, \mathbf{z}, \ldots$, for sequences $\mathbf{x} = (x_n), \mathbf{y} = (y_n), \mathbf{z} = (z_n), \ldots$ of points in X.

Recall that a subset E of a metric space (X, d) is called bounded if

$$\delta(E) = \sup\{d(a, b) : a, b \in E\} \leq M,$$

where M is a positive real constant number. A subset A of a metric space X is said to be an ε-net in X if

$$X = \bigsqcup_{a\in A} B(a, \varepsilon).$$

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