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On strongly Bazilevic functions associated with generalized Robertson functions

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ABSTRACT

In this paper, we define a class $\widetilde{B}_k(\alpha, \rho, \beta, \gamma)$ of analytic functions by using a generalized Robertson function which generalizes a number of classes studied earlier such as the class of strongly Bazilevic functions. Some interesting properties of this class, including coefficient difference problems, arc length and a sufficient condition for univalency, are investigated.

1. Introduction

Let $P_k^\alpha(\rho)$ be the class of functions $p(z)$ analytic in the open unit disc $E = \{z : |z| < 1\}$ such that $p(0) = 1$ and satisfying the condition

$$\int_0^{2\pi} \left| \left\{ \frac{\operatorname{Re} e^{i\alpha} p(z) - \rho \cos\alpha}{1-\rho} \right\} \right| d\theta \le k\pi \cos\alpha, \quad z = re^{i\theta},$$

where $k \ge 2$, $0 \le \rho < 1$, α is real, $|\alpha| < \frac{\pi}{2}$. For $k = 2$, $\alpha = 0$ and $\rho = 0$, the class $P_k^\alpha(\rho)$ reduces to the class P of functions $p(z)$ analytic in E with $p(0) = 1$ and whose real part is positive.

Let $V_k^\alpha(\rho)$ be the class of functions $f(z)$ analytic in E with $f(0) = 0, f'(0) = 1$ and satisfying the condition

$$1 + \frac{zf''(z)}{f'(z)} \in P_k^\alpha(\rho). \tag{1.1}$$

When $\rho = 0$ and $\alpha = 0$, we obtain the class $V_k (k \ge 2)$ of functions with bounded boundary rotation. The class $V_k^\alpha(\rho)$ was introduced and studied in some detail in [1]. It can easily be shown that $f(z) \in V_k^\alpha(\rho)$ if and only if there exists $f_1(z) \in V_k$ such that

$$f'(z) = (f_1'(z))^{(1-\rho)e^{-i\alpha}\cos\alpha}. \tag{1.2}$$

We now define a class $\widetilde{B}_k(\alpha, \rho, \beta, \gamma)$ of analytic functions by using the class $V_k^\alpha(\rho)$ as follows.

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