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# Constructing efficient substructure-based preconditioners for BEM systems of equations ☆

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In this work, a generic substructuring algorithm is employed to construct global block-diagonal preconditioners for BEM systems of equations. In this strategy, the allowable fill-in positions are those on-diagonal block matrices corresponding to each BE subregion. As these subsystems are independently assembled, the preconditioner for a particular BE model, after the **LU** decomposition of all subsystem matrices, is easily formed. So as to highlight the efficiency of the preconditioning proposed, the Bi-CG solver, which presents a quite erratic convergence behavior, is considered. In the particular applications of this paper, 3D representative volume elements (RVEs) of carbon-nanotube (CNT) composites are analyzed. The models contain up to several tens of thousands of degrees of freedom. The efficiency and relevance of the preconditioning technique is also discussed in the context of developing general (parallel) BE codes.



## 1. Introduction

Applying iterative solvers to large-order engineering problems has been intensively pursued in the last decades, mainly because their unquestionable appeal to solve truly large models [1,2]. Herein, the parallelism embedded in them allied with the today's parallel computer architectures plays a decisive role, so that it can be well stated that developing fast scalable (preconditioned) parallel Krylov solvers is a key point for getting high-fidelity solution for large-order complex engineering problems. In these cases, direct solvers may be exceedingly expensive concerning both memory and CPU time, and their parallel implementation is awkward.

For general non-symmetric matrices, like BE matrices, based on the number of terms involved in the iterative formulas, the Krylov solvers can be subdivided in two broad classes of algorithms: long-recurrence algorithms (GMRES and variants) and short-recurrence ones (Bi-CG and variants). Over the last several decades, milestone contributions in these algorithms have been definitely given by the following works: the Lanczos method (by Lanczos in 1952) [3], the Bi-CG method (by Fletcher in 1976) [4], the GMRES method (by Saad and Schultz in 1986) [5], the CGS method (by Sonneveld in 1989) [6], the Bi-CGSTAB (by van der Vosrt in 1992) [7], and the Bi-CGSTAB($l$) (by Sleijpen and Fokkema in 1993) [8]. Of course, in this period of time, a series of other works that significantly contributed for increasing the efficiency of Krylov solvers have also been published, including those related to particular applications to symmetric definite matrices.

Particularly for BEM systems of equations, the first successful applications of iterative solvers were reported at the end of the 80s and beginning of the 90s [9–12], wherein diagonal-preconditioned Bi-CG [9–10,12], and preconditioned GMRES [11] methods were used. According to the authors' knowledge, before these works, only basic iterative methods as the Jacobi or Gauss–Seidel methods, or at most the CGN solver, which consists of applying the CG method to the normal equations, $\mathbf{A}^T\mathbf{A}\mathbf{x}=\mathbf{A}^T\mathbf{b}$, had been considered [13–14]. A patent disadvantage of these iterative solvers are the non-reliability regarding convergence, so that they actually cannot be regarded as general-purpose solvers for practical applications. In fact, applying basic iterative methods, convergence is assured only if the spectral radius of the corresponding iteration matrix is less than 1, which is not the case for general systems. On the other hand, considering the CGN has the disadvantage of squaring the condition number of the original system $\mathbf{A}\mathbf{x}=\mathbf{b}$, which may cause the iterative process fail to converge. The Bi-CG and GMRES methods, and their variants (or combinations) are then the remaining alternatives for deriving general-purpose solvers for BEM equations.

In fact, long-term recurrence methods as GMRES and variants should be avoided because of memory requirements for large problems and non-rare convergence stagnation in practice.

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