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The localized differential quadrature method for two-dimensional stream function formulation of Navier–Stokes equations

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ABSTRACT

Localized differential quadrature (LDQ) method is employed to solve two-dimensional stream function formulation of incompressible Navier–Stokes equations. Being developed by introducing the localization concept to the general differential quadrature (GDQ) method, the employment of LDQ method becomes efficient and flexible, especially for the simulations of large scale computations. By introducing the Lagrange stream function to vorticity transport equation, the governing equation—the fourth-order partial differential equation (PDE)—is derived. To stably obtain the solutions of the fourth-order PDE, a fictitious point method is included to treat the boundary conditions. To examine the present scheme, two different types of classic benchmark fluid flow problems are proposed, including driven cavity flow problems and backward-facing step flow problems. The good agreement of solutions demonstrate the robustness and feasibility of the proposed scheme. Conclusively, the LDQ method is sufficient and appropriate enough to simulate the solutions of stream function formulation of Navier–Stokes equations with various Reynolds numbers.

1. Introduction

Differential quadrature (DQ) method is first presented by Bellman et al. [1,2], and the main idea is originated from the concept of integral quadrature. Through expressing a derivative as a linear weighted sum of all functional values along the direction of its respective coordinate and solving the resulting matrix, the weighting coefficients of the derivative can be determined. After determining the weighting coefficients, the solution at the interested mesh point can be accurately obtained even though few computational points are employed. The resulting matrix, however, formed for obtaining the weighting coefficients is easy to be ill-conditioned, and thus the total amount of grid points for each direction usually cannot exceed 13 [3]. To improve the feasibility of the DQ method, Quan and Chang [4,5] derived explicit formulations to calculate the weighting coefficients of the derivatives of first- and second-order through selecting the test function as the Lagrange interpolation polynomials. Moreover, because the adoption of the Lagrange polynomials, the distribution of mesh grids is unnecessary to be uniform, and thus it is allowed to adjust the mesh to physically fit to the predicted distribution of real field. For a boundary layer problem, fining the size of mesh grids near the boundary can improve the scheme to accurately catch the rapidly variation near the boundary within few grid points. In contrast to finite difference method (FDM), the distribution of mesh grids is more flexible, and the simulation of the problem is more efficient. Nevertheless, the employment of DQ method is still confined to the problem which is governed by first- and second-order partial differential equations, because the determination of the weighting coefficients for higher order derivatives is quite complicated. Until 1990, Shu and Richard [6] presented a general DQ (GDQ) method, by calculating a recurrence relationship, the weighting coefficients of arbitrary-order derivatives can be effortlessly obtained.

Shu et al. [7,8] utilized the GDQ schemes to simulate the solutions of two-dimensional incompressible Navier–Stokes equations. Lo et al. [9] adopted the GDQ to solve velocity–vorticity formulation of Navier–Stokes equations for 3D natural convection problem. In contrast to the applications of other numerical methods, however, the allowed amount of mesh grids for GDQ method is still limited. The reason is that the resulting matrix is easy to be ill-conditioned as a large amount of mesh grids is used. In addition, the requirement of storage for resulting matrix becomes quite large because the weighting coefficients are calculated by considering the whole computational points along the direction of its derivative. In other words, the amount of mesh grids is not allowed to be large when GDQ scheme is employed. Therefore, by locally choosing the reference points, a groundbreaking concept—localization—is proposed by Shu [3] to overcome the defect of the application of GDQ method, and the method is the so-called localized differential quadrature (LDQ) method. By locally

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