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Elastic analysis of a half-plane containing an inclusion and a void using a mixed volume and boundary integral equation method

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ABSTRACT

A mixed volume and boundary integral equation method is used to calculate the plane elastostatic field in an isotropic elastic half-plane containing an isotropic or anisotropic inclusion and a void subject to remote loading parallel to a traction-free boundary. A detailed analysis of the stress field is carried out for three different geometries of the problem. It is demonstrated that the method is very accurate and effective for investigating local stresses in an isotropic elastic half-plane containing multiple isotropic or anisotropic inclusions and multiple voids.

1. Introduction

Stress analysis of heterogeneous solids generally requires the use of numerical approaches based on the finite element or boundary integral equation formulations. Finite element method presents difficulties in dealing with problems involving extended (infinite or semi-infinite) media. Boundary integral equation method also becomes cumbersome in dealing with problems containing anisotropic inclusions. It has been demonstrated that the volume integral formulation can overcome both of these limitations in heterogeneous problems involving infinite or semi-infinite media [1,2].

This paper is concerned with the analysis of the stress field in a heterogeneous elastic solid containing an arbitrary distribution of multiple circular inclusions and voids embedded in a half-plane using the mixed volume and boundary integral equation method. The inclusions may be either isotropic or anisotropic, but the matrix is assumed to be isotropic. Of particular interest here is the influence of the free surface of the half-plane and geometry of inclusions and voids on the local stresses near the inclusions.

For this problem, the VIEM is effective for problems with anisotropic inclusions due to the fact that only Green's function for the unbounded isotropic matrix is needed in the formulation. However, problems involving voids can be most effectively solved by the boundary integral equation method [3–6]. Thus it is advantageous to apply a combination of the two methods so that both of these conditions are automatically satisfied in the formulation [7,8].

The problem of a single circular inclusion in an elastic half-plane has been examined by several authors (e.g., Melan [9], Richardson [10], Saleme [11], Shioya [12], Lee et al. [13] and Al-Ostaz et al. [14]). Likewise, problems associated with multiple inclusions in an elastic half-plane have also been investigated by several authors (e.g., Dong et al. [15], Legros et al. [16] and Kushch et al. [17]).

The problem of a single void in an elastic half-plane has been investigated by several authors, including Jeffery [18] and Mindlin [19]. However, Jeffery's solution was incorrect and Mindlin's solution for the hoop stress at a hole breaks down as the hole approaches the boundary of the half-plane. Pobedonostsev [20] presented the solution of the more general problem of the half-plane containing a circular hole under the action of arbitrary distributed tractions on the boundary. Duan et al. [21] used a complex integral equation method to study interaction problems between holes and other defects in an infinite or semi-infinite elastic plane. They investigated the asymptotic behavior of the stress distribution in neighboring defects. Callias and Markenscoff [22] analyzed the hoop stress at a hole near the edge of a plate

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