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A numerical algorithm to damp instabilities of a retarded potential integral equation

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ABSTRACT

This paper deals with a new systematic approach of the instability problem in acoustic boundary elements models. When a long enough marching-on-time (MOT) computation is carried out, an exponential growth of the pressure magnitude often appears. This unrealistic behaviour is a serious drawback for the method of retarded potential integral equations. In this paper, we show that the instability problem of this type of algorithm can be translated in terms of a polynomial eigenvector problem (PEVP). In the case of the hypersingular derivative Green method, examples are shown of suppressing all numerical instabilities related to open or closed diffracting objects using the PEVP. Among them, the vibroacoustic model of a test structure used by the CNES (Centre National d'Etudes Spatiales) is successfully processed.

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1. Introduction

1.1. Motivation

The instability of the calculations in time integral formulation has been documented for a long time. It is evidenced by oscillations of increasing amplitude, often with a $2\Delta t$ period, (Δt being the time step), sometimes with a longer period. It has been analyzed as reflecting the growth of mesh modes, due to unavoidable numerical errors [1]. Our experience is the following: when using the "normal derivative of the Kirchhoff integral equation" (also called the "thin" element or "shell-type" element representation [2,3]), a particularly well-suited formulation for industrial vibroacoustic situations, there is a chance of discovering an unexpected instability after the computation ends. The longer the time span of the computation, the higher the risk of revealing an instability corrupting the result. The instable components indeed become more and more obvious as the computation runs forward. Some instability modes indeed grow very slowly, and they appear after a great many time steps. This was observed on a "pneumatic" non-oscillatory instability affecting closed volumes [3]. But a basic use of the computing code shows that oscillatory instabilities are also of concern. The reliability of the method would be much more established, if we had a technique available for detecting and correcting the instability before the computation begins, how faint it may be.

It could be argued that the frequency representation is formally equivalent to the time representation, and then it should be preferred as stable by nature. This argument is not decisive. In applications, the acoustic integral model may have to be coupled to non-linear elements. These elements are mechanical (for example: light panels or membranes), or acoustical (for example: convergent channels connected to high pressure level areas). Non-linear control systems also may be of concern.

The work presented here refers principally to the "thin" element representation, as programmed in the vibroacoustic software ASTRYD [4]. The "hypersingularity" called "Hadamard's finite part" is then present in the algorithm [3].

1.2. State of the art in instability of time integral formulations

There are very few publications on time hypersingular formulations. However, papers by Ergin et al. [5,6] are to be noted. The bibliography research has to be extended to other, non-hypersingular, formulations.

The high-frequency oscillating instabilities have been well studied in the electromagnetic domain, where this technique is frequently used. The methodological contributions of Rynne and Smith [1] and Davies and Duncan [7,8] are to be noted: a theoretical study based on the Von Neumann's stability analysis technique, which is usual in finite differences, is possible in simplified geometries, planes or spheres, for every mesh wavenumber value. The aforementioned authors have then searched for the numerical propagation modes in the scheme. Their theoretical predictions on more or less filtering explicit schemes are in line with their numerical experiments. Unfortunately the analysis of implicit schemes has not been developed. Works of Gaul

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