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Group preserving scheme for the Cauchy problem of the Laplace equation

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ABSTRACT

In this paper, we consider the Cauchy problem for the Laplace equation by group preserving scheme (GPS) which is an ill-posed problem, because the solution does not depend continuously on the data. For this, the Laplace equation, by using a semi-discretization method namely method of line, is converted to an ODEs system and then obtained ODEs system is considered by GPS. Stability of GPS for ill-posed Laplace equation is shown. The problem numerical results show the efficiency and power of this method.

1. Introduction

In this paper, we consider the following Laplace equation:

$$u_{xx}+u_{yy}=0, \quad 0<x<\pi, \ 0<y<1, \tag{1.1}$$

$$u(x,0)=\phi(x), \quad 0\le x\le \pi, \tag{1.2}$$

$$u_y(x,0)=0, \quad 0\le x\le \pi, \tag{1.3}$$

$$u(0,y)=u(\pi,y)=0, \quad 0\le y\le 1. \tag{1.4}$$

Small perturbations of $\phi(x)$ in (1.2) may cause a large errors in the solution $u(x,y)$ for $0<y\le 1$. It can be verify that

$$u(x,y)=\frac{\lambda}{k^i}\sin(kx)\cosh(ky) \tag{1.5}$$

is the exact solution of problem (1.1)–(1.4) whenever

$$\phi(x)=\frac{\lambda}{k^i}\sin(kx), \tag{1.6}$$

where $k,\ i\in\mathbb{Z}^+$ and $\lambda\in\mathbb{R}\backslash\{0\}$. Although $\sup_{0<x<\pi}|\phi(x)|\to 0$ as $k\to\infty$, for fixed $y>0$ we have $\sup_{0<x<\pi}|u(x,y)|\to\infty$ as $k\to\infty$. Thus, the Laplace equation is a severely ill-posed problem and its solving using classical numerical methods is impossible and requires special techniques such as regularization [1]. This paper deals with some accurate method without any regularization. In this sense, Hon and Wei in [2] have transformed the Laplace equation to a classical moment problem and considered its convergency and stability estimates based on the Backus–Gilbert algorithm. Kubo in [3], has obtained an L^2-estimate for the Cauchy problem for the Laplace equation. Stability results for the Laplace equation has been considered by Hào and Hein [4], when

$$\phi\in L_p(\mathbb{R}), \quad u(.,y)\in L_p(\mathbb{R}), \quad \|u(.,y)\|_{L_p(\mathbb{R})}\le M<\infty, \quad p\in[1,\infty].$$

The error between the continuous Laplace problem and the difference approximation obtained via a suitable minimization problem has estimated by a discretization and a regularization term by Reinhardt and et al. [5]. Qiu and Fu, used the wavelet regularization method to restore the stability of the solution of two-dimensional Laplace equation in the strip $0<x\le 1$ [6]. Numerical solutions of Laplace equation with Dirichlet and Neumann boundary conditions has been obtained by HAM in [7]. Also, Sadighi and Ganji, have used the homotopy-perturbation and Adomian decomposition methods to obtain the solutions of Laplace equation with Dirichlet and Neumann boundary conditions [8]. Meyer wavelet transform as a regularization procedure to restore the stability of the solution has been applied by Vani et al. [9] for Laplace equation and they have shown that under certain conditions this regularized solution is convergent to the exact solution when a data error tends to zero. Cheng et al. [10] added a fourth-order mixed derivative term to the Laplace equation and studied it with some error stability estimates for the flux. Some other works about the Cauchy problem of the Laplace equation can be found in [11–16].

In this paper, firstly we use the method of line for semi-discretization of Laplace equation [17,18] and then we apply The GPS, which firstly derived by Liu [19]. GPS uses the Cayley transformation and the Padé approximations in the augmented space, namely Minkowski.

After that, Liu [20] applied the group preserving scheme (GPS) for backward heat conduction problems, which is an ill-posed problem and considered its stability and showed that obtained

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