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Dispersion and pollution of the improved meshless weighted least-square (IMWLS) solution for the Helmholtz equation

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ABSTRACT

The meshless weighted least-square (MWLS) method is a meshless method based on the moving least-square (MLS) approximation. Compared with the Galerkin based meshless methods, the MWLS avoids numerical integrations, which improves the computational efficiency significantly. The MLS may form ill-conditioned system of equations, an accurate solution of which is difficult to obtain. In this paper, by using the weighted orthogonal basis function to construct the improved moving least-square (IMLS) approximation and the Lagrange multiplier method to enforce the Dirichlet boundary condition, we derive the formulas and perform the dispersion analysis for an improved meshless weighted least-square (IMWLS) method for two-dimensional (2D) Helmholtz problems. Results demonstrated that the IMWLS is more accurate and has advantages in handling dispersion. A 2D industrial model problem illustrated that the proposed method can easily reach higher frequency without losing accuracy.

1. Introduction

Accurate and efficient numerical simulation of acoustic problems governed by the Helmholtz equation is still an open problem especially for medium frequencies. By using the standard finite element method (FEM), computational accuracy decreases quickly with increasing wave numbers, while the boundary element method (BEM) suffers from high computational costs because of the full and non-symmetric algebraic equations to be solved.

As we know, the main reason for the pollution error is that the numerical wave number does not match the exact wave number which is also called ‘dispersion’. In order to depress the dispersion for high wave numbers, highly refined finite element meshes (*h*-FEM) or higher orders of polynomial approximation (*p*-FEM) are required, and the *hp*-FEM [1] seems to give good results. However, to obtain an acceptable level of accuracy, more than ten elements per wavelength are required [2]. For large wave number, refining the mesh to maintain this requirement may become prohibitively expensive. Several methods to stabilize the FEM have also been developed, such as the Galerkin least-square (GLS) FEM [3], the quasi-stabilized FEM (QSFEM) [4], the residual-free FEM (RFFEM) [5] and the residual-based FEM [6] with applications to the Helmholtz problem. A review of these methods can be seen in [7,8]. However, none of them eliminates the dispersion.

Meshless methods have several advantages over the classical mesh-based methods. It has already been shown that the classical element-free Galerkin method (EFGM) [9] gives very accurate results for interior acoustic problems and dramatically reduces the dispersion effect compared with the standard FEM [10]. The multiple-scale reproducing kernel particle method (RKPM) [11] and the so-called radial point interpolation method (RPIM) [12], give very accurate results for interior Helmholtz problems too. Unfortunately in order to ensure their accuracy, delicate background cells and a large number of quadrature points have to be used for the global numerical integration for the Galerkin method (the EFGM, the RKPM and the RPIM), which dramatically increases the computational cost. Recently two methods based on the method of fundamental solutions (MFS) [13] and the boundary knot method (BKM) [14] have been extended to Helmholtz-type equations. Both the two methods, however, need to use the inner nodes for inhomogeneous problems to guarantee the stability and accuracy of the solution. More recently the boundary-node method (BNM) [15] has been applied to Helmholtz problems which showed high convergence rates and high accuracy. However, it is difficult to satisfy the boundary conditions accurately in BNM. This makes it computationally much more expensive than the BEM [16].

A meshless weighted least-square (MWLS) method has been proposed to solve problems of elastostatics [17], and the advantages of better accuracy, high efficiency and fast convergence,

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