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Scalable total BETI based solver for 3D multibody frictionless contact problems in mechanical engineering

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ABSTRACT

A total BETI (TBETI) based domain decomposition algorithm with the preconditioning by a natural coarse grid of the rigid body motions is adapted for the solution of multibody frictionless contact problems of linear elastostatics and proved to be scalable, i.e., the cost of the solution is asymptotically proportional to the number of variables. The analysis admits floating bodies. The proofs combine the original results by Langer and Steinbach on the scalability of BETI for linear problems and our development of optimal quadratic programming algorithms for bound and equality constrained problems. The theoretical results are verified by numerical experiments. The power of the method is demonstrated on the analysis of ball bearings.

1. Introduction

The contact of one body with another is a typical way how loads are delivered to a structure and at the same time it is the mechanism which supports structures to sustain the loads. Thus we do not exaggerate much if we say that the contact problems are in the heart of mechanical engineering. Solving large multibody contact problems of linear elastostatics is complicated by the inequality boundary conditions, which make them strongly non-linear, and, if the system of bodies includes "floating" bodies, by the positive semi-definite stiffness matrices resulting from the discretization of such bodies. Observing that the classical Dirichlet and Neumann boundary conditions are known only after the solution has been found, it is natural to assume the solution of contact problems to be more costly than the solution of a related linear problem with the classical boundary conditions. Since the cost of the solution of any problem increases at least linearly with the number of the unknowns, it follows that the development of a scalable algorithm for contact problems is a challenging task which requires to identify the contact interface in a sense for free. Let us recall that an algorithm is *numerically scalable* if the cost of the solution increases nearly proportionally to the number of unknown variables, and it enjoys the *parallel scalability* if the cost of the solution can be reduced nearly proportionally to the number of available processors. Scalable algorithms are also called optimal.

In spite of this, there has been made a considerable effort in this direction and a number of interesting results have been obtained. Most of the results, either experimental or theoretical, were obtained for the problems discretized by the finite element method (FEM), either in the framework of the domain decomposition methods, see, e.g., Schöberl [42], Dureisseix and Farhat [22], Avery et al. [1], Avery and Farhat [2], Dostál and Horák [19], or by the multigrid methods, see, e.g., Kornhuber [34], Kornhuber and Krause [35], and Wohlmuth and Krause [54]. Most recently, a theoretically supported scalable algorithm for both coercive and semi-coercive contact problems was presented by Dostál et al. [16].

A number of researchers developed effective algorithms also for the solution of contact problems by the boundary element method (BEM), including problems with friction [23], problems discretized by mortars [43], or semi-coercive problems [15]. The main benefit of the application of the BEM, as compared with the more popular FEM, is that the formulation of the problem is reduced to the boundary of the underlying domain which yields a significant dimension reduction. In particular, BEM is desirable, e.g., when dealing with large or unbounded domains [39] or shape optimization problems [9]. However, since BEM requires the explicit knowledge of a fundamental solution of a given partial differential operator, it is applicable only to the problems involving materials with rather simple properties.

The activity in the development of optimal algorithms for contact problems discretized by BEM lagged behind the related FEM research. This is rather surprising, since BEM seems to be a suitable tool for treating the non-linearity which is restricted to the contact interface. The reason for such a delay is that a suitable boundary element domain decomposition method, BETI (boundary element tearing and interconnecting), was introduced by Langer and Steinbach [38] only in 2003, much later than Farhat and Roux [26] proposed their FETI (finite element tearing and interconnecting)

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