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A new semi-analytical method with diagonal coefficient matrices for potential problems

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ABSTRACT

In this paper, a new semi-analytical method is proposed for solving boundary value problems of two-dimensional (2D) potential problems. In this new method, the boundary of the problem domain is discretized by a set of special non-isoparametric elements that are introduced for the first time in this paper. In these new elements, higher-order Chebyshev mapping functions and new special shape functions are used. The shape functions are formulated to provide Kronecker Delta property for the potential function and its derivative. In addition, the first derivative of shape functions are assigned to zero at any given control point. Finally, using weighted residual method and implementing Clenshaw–Curtis quadrature, the coefficient matrices of equations system become diagonal, which results in a set of decoupled governing equations for the whole system. This means that the governing equation for each degree of freedom (DOF) is independent from other DOFs of the domain. Validity and accuracy of the present method are fully demonstrated through four benchmark problems.

1. Introduction

A potential problem involves one of the most important partial differential equations as it occurs in pollutant diffusion problems, electrostatics and gravitational fields, and incompressible fluid flow and steady-state heat conduction problems [1]. From mathematical point of view, well-known potential problems include Laplace or Poisson equation in a specific domain with a number of boundary conditions. From physical viewpoint, space coordinates are the sole coordinates of the problem that is independent of time variable.

Potential problems may be analytically solved when the geometry of the problem is simple. In cases of complex geometry and/or boundary conditions, these problems should be solved using numerical methods.

Various types of numerical methods such as finite element method (FEM), boundary element method (BEM), scaled boundary finite element method (SBFEM), and meshless methods are commonly used to model potential problems. The use of FEM is advantageous as the procedures are versatile in nature and well-established [2]. Alternatively, BEM requires substantially reduced surface discretizations, and is appealing an alternative to FEM for potential problems [3–11]. BEM does not require the domain of the problem to be discretized. Consequently, fewer unknowns are required to be stored, which leads to a saving in storage space and CPU time. BEM requires a fundamental solution to the governing differential equation in order to obtain the boundary integral equation. In other words, BEM needs fundamental solutions that are dependent on the problem of interest. Although the coefficient matrices of BEM are much smaller than those of FEM, they are usually non-symmetric, non-positive definite, and fully populated.

Combining the advantages of FEM and BEM, SBFEM has been successfully developed [12]. SBFEM discretizes only the boundary of the domain of interest with surface finite elements by transforming the governing partial differential equations to a system of ordinary differential equations, which may be solved analytically. SBFEM, which needs no fundamental solution as for BEM, have also been used for the analysis of potential problems (see, for example [13–15]).

During the last decade, researchers have paid attention to meshless methods, which are the mesh reduction methods with no mesh requirements and only boundary nodes are necessary. Several meshless methods have been reported in the literature among which, various element-free methods [16–18], various boundary node methods [19–22], boundary face method [23], boundary point method [24], local boundary integral equation [25], and coupled FEM and meshless local Petrov–Galerkin method [26] have been employed in the analysis of potential problems.

In this paper, two-dimensional (2D) potential problems are investigated using a novel semi-analytical method. In this new

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