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Evaluation of free terms in hypersingular boundary integral equations

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SUMMARY

The accurate numerical solution of hypersingular boundary integral equations necessitates the precise evaluation of free terms, which are required to counter discontinuous and often unbounded behaviour of hypersingular integrals at a boundary. The common approach for the evaluation of free terms involves integration over a portion of a spherical shaped surface centred at a singularity and allowing the radius of the sphere to tend to zero.

In this paper two alternative methods, which are shape invariant, are proposed and investigated for the determination of free terms. One approach, the *point-limiting method*, involves moving a singularity towards a shrinking integration domain at a faster rate than the domain shrinks. Issues surrounding the choice of approach and shrinkage rates, and path dependency are examined. A related approach, the *boundary-limiting method*, involves moving an invariant but shrinking boundary toward the singularity again at a faster rate than the shrinkage of the domain. The latter method can be viewed as a vanishing exclusion zone approach but the actual boundary shape is used for the boundary of the exclusion zone. Both these methods are shown to provide consistent answers and can be shown to be directly related to the result obtained by moving a singularity towards a boundary, i.e. by comparison with the direct method. Unlike the spherical approach the two methods involve integration over the actual boundary shape and consequently shape dependency is not a concern. A particular highlight of the point limiting approach, as a result of field approximations being restricted to the boundary, is the ability to obtain free terms in a mixed formulation without reference to the underpinning constitutive equations, which is not available to the spherical method.

Focus in the paper is on the 2-D potential equation as this is shown to be sufficient to demonstrate the concepts involved.

1. Introduction

Research into the development and application of hypersingular boundary integral equations has been ongoing over the past decade. The approach presents an alternative to the general solution of thermal, elastostatic and elastodynamic problems. However, it is invariably more costly computationally than the standard integral equation formulation involving kernels with greater complexity and higher-order singularities. The method is often employed in dual formulations in combination with standard integral methods; a common usage is fracture mechanics for the prediction of stress intensity and crack propagation [1–5].

The hypersingular approach is formulated as the sum of free terms and singular integrals incorporating two-point kernels. A number of investigations have postulated the existence of additional free terms [6–9] associated with corners connected to adjacent curved boundary parts. Free terms and associated integrals typically exist in the sense of the Hadamard finite part whose determination involves the creation of a vanishing exclusion zone and asymptotic analysis [10–16]. A reasonable review of the analytical treatments proposed for the evaluation of hypersingular integrals is given in Ref. [9].

The "natural" shape of the exclusion zone is a ε-ball $B_\varepsilon(\mathbf{x}) = \{\mathbf{y} : \|\mathbf{y}-\mathbf{x}\|_2 < \varepsilon\}$ whose boundary is an ε-sphere $S_\varepsilon(\mathbf{x}) = \{\mathbf{y} : \|\mathbf{y}-\mathbf{x}\|_2 = \varepsilon\}$ oriented by the normal vector $\mathbf{n}(\mathbf{y})$ pointing to the centre $\mathbf{x}$ [10]. However, it is recognised that the Hadamard finite part is not unique and depends on the shape of the vanishing exclusion zone [10,11]. A question that immediately arises: does the non-uniqueness lead to incorrect free terms in the governing integral equation? Shape dependency is a feature of the individual integrals in an integral equation and for these individual integrals the application of the spherical approach can differ with the result obtained using a direct approach [11]. However, despite these differences, integral equations contain combinations of said integrals which are shape independent giving rise to correct free terms. This paper re-examines the issue of shape dependency with the introduction of two new limiting processes for the evaluation of free terms in the hypersingular boundary integral equation for the

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