



Automatic particular solutions of arbitrary high-order splines associated with polyharmonic and poly-Helmholtz equations

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ABSTRACT

The explicit analytical particular solutions of the splines and monomials for the polyharmonic and poly-Helmholtz operators and their products were derived in the author's previous study. These solutions enable an implementation for automatically approximating particular solutions of arbitrary high-order splines. The automatic particular solutions obtained by the proposed implementation are extremely accurate. In the case of a polyharmonic equation, these solutions are more accurate than the numerical solutions obtained using the multiquadrics within the limits of the IEEE double precision. In the case of poly-Helmholtz and product equations, this implementation can also result in very accurate solutions despite the fact that an analytical particular solution for the multiquadrics does not exist. After particular solutions are obtained, boundary-type numerical methods, such as the boundary element method, the method of fundamental solutions and the Trefftz method, can be applied to solve the homogeneous differential equations.

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1. Introduction

Boundary-type numerical methods, such as the boundary element method (BEM), the method of fundamental solutions (MFS) and the Trefftz method (TM), have been important research topics in the field of computing for many years, since these methods can reduce the dimensionality of a considered problem by one. A comprehensive review of the boundary-type numerical methods can be found in the article of Cheng and Cheng [1]. However, when these methods are used for solving an inhomogeneous partial differential equation, a considerable amount of computational effort is required for evaluating the particular solution associated with the forcing term. Therefore, the dual reciprocity method (DRM) was first proposed by Nardini and Brebbia [2] for transferring the domain integral of the BEM by interpolating the forcing term with a series of global basis functions. Then, the DRM was successfully used for solving a wide variety of problems in the fields of engineering and science [3].

In the early development of the DRM during the period of 1982–1990, an ad-hoc trial function $1+r$ was exclusively chosen for interpolating the forcing term without obvious reasons. Therefore, Golberg and Chen [4] suggested that the well-established theory of radial basis functions (RBFs) should be used as a mathematical foundation for the selection of the trial functions of the DRM. Among these trial functions, Duchon's polyharmonic splines (PSs) [5] and Hardy's multiquadrics (MQ) [6] are the most popular. Golberg [7,8], Chen [9] and Karur and Ramachandran [10] demonstrated that the PS is superior to the ad-hoc trial function. On the other hand, Golberg et al. [11] further improved the accuracy of the

approximated particular solution by utilizing the exponential convergence rate of the MQ. In 1998, the improvement using these new RBFs was reviewed by Golberg and Chen [12].

Then, the development of the DRM was basically redirected to the search of analytical particular solutions of PSs associated with various operators. Muleshkov et al. [13] found a particular solution of the Helmholtz operator. In 2000, Cheng [14] reviewed the issue by deriving analytical particular solutions of Laplacian, Helmholtz-type and polyharmonic operators. Furthermore, Muleshkov and Golberg [15] derived an analytical solution of the multi-Helmholtz-type equation. Recently, Tsai et al. [16] generalized the derivation of analytical particular solutions to poly-Helmholtz operators and their products. In addition to considering a single operator, Cheng et al. [17] found analytical particular solutions of thermoelasticity by solving coupled PDEs.

On the other hand, there was another parallel development for obtaining particular solutions using the Chebyshev polynomials. Golberg et al. [18] obtained extremely accurate particular solutions by approximating the forcing term in a rectangular domain that was sufficiently large to enclose the computational domain. However, some tedious bookkeeping was required in their study. Reutskiy and Chen [19] remedied the tedious bookkeeping by using two-stage approximations of the trigonometric functions and Chebyshev polynomials. On the other hand, Karageorghis and Kyza [20] studied the same issue by directly approximating the particular solutions using Chebyshev polynomials. Recently, Ding et al. [21] implemented a recursive scheme to obtain the particular solutions of Chebyshev polynomials without the requirement of bookkeeping. Alternatively, Tsai [22] derived explicit analytical particular solutions for arbitrary high-order monomials and used a floating number computation method for solving the polyharmonic and poly-Helmholtz equations.

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