



# Improving the stability of time domain dual boundary element method for three dimensional fracture problems: A time weighting approach

Amin Kamali Yazdi<sup>a</sup>, Babak Omidvar<sup>b,\*</sup>, Mohammad Rahimian<sup>a</sup>

<sup>a</sup> Department of Civil Engineering, Faculty of Engineering, University of Tehran, Tehran, Iran

<sup>b</sup> Department of Natural Disaster Management, Faculty of Environment, University of Tehran, Tehran P.O. Box 14155-6135, Iran

## ARTICLE INFO

### Article history:

Received 9 January 2011

Accepted 11 May 2011

Available online 15 June 2011

### Keywords:

Dual boundary element method

Stability

Three dimensional problems

Time weighting

## ABSTRACT

This paper deals with the stability of time domain dual boundary element method (DBEM). A time-weighted time domain DBEM is presented in this study and used for the first time in order to improve the stability of the standard time domain dual boundary element method. In this research a time weighting function with a prediction algorithm based on constant velocity algorithm has been utilized. The present approach was tested for three-dimensional fracture problems. The computer cost for the time of the presented approach is very close to the standard form. The results of numerical experiments carried out within this study indicate that the time weighting method, which is suggested for time domain DBEM, has more stability in comparison with the conventional method.

© 2011 Elsevier Ltd. All rights reserved.

## 1. Introduction

There are some reports among the published articles in transient elastodynamics in which numerical response has been disturbed either due to the presence of noise or a consequence of unstable behavior. Instability of the transient dynamic BEM formulation, which causes a building up of errors as the time stepping progressed, was studied by Cole et al. [1]. Studies of Mansur [2] on a one-dimensional rod under a Heaviside-type forcing function showed that time steps smaller than  $0.6l/c$  ( $l$  is the element length and  $c$  is the wave propagation velocity) may make an excessive noise, which could lead to instability; on the other hand, because of violation of the causality condition, large time steps may cause errors. Hence the time step value must be within an interval whose lower and upper limits are specified by the problem being studied and the space discretization adopted. So, in analyses that the mesh contains elements with unequal lengths, choosing an optimum time step could be difficult.

These days, there are techniques that can lead to stable time domain BEM algorithms. The QUOTE scheme and the half-step approach introduced by Siebrits and Peirce [3], the  $\theta$ -scheme presented by Yu et al. [4–6] and the time-convolution modification proposed by Soares and Mansur [7,9] and Soares [8] are some of the related works. Employing time and space weighting functions is another approach to improve stability [10,11].

The integral equations for a time-weighted time domain BEM were presented by Maier et al. [12]. However in this study only the theoretical aspects and neither numerical implementation nor

application was taken into account. In time-weighting methods the average of whole time history of the solution is considered in a weighted residual sense. As a result, that although has no significant effect on early stages, it may affect greatly on the late time response, especially in problems with unstable behavior. Yu et al. [10] applied time weighting method for BEM in 2D scalar wave problems.

The dual boundary element method developed by Portela et al. [13,14] for 2D problems and Mi and Aliabadi [15] for 3D problems has been shown to be general, and computationally an efficient way for modeling of crack problems in BEM. Applying the displacement equation for collocation node on one of the crack faces, and the traction equation on the other two independent boundary integral equations was incorporated in the DBEM. As can be seen in the present study, dual boundary element method has difficulty with instability, and this article deals with 3D DBEM and its instability.

The study of 3D transient elastodynamic problems with actual non-zero values of the Poisson's ratio is considered as one of the most challenging situations from the stability point of view of time domain BE approaches. In those cases, the existence of two kinds of waves in the fundamental solution and the reflection of different kinds of waves on the boundaries may cause causality errors and instability problems. Most published works in the field of stability improvement deal with two-dimensional models and they rarely involve with three-dimensional models with non-zero Poisson's ratio.

In this paper a weighted formulation is applied, which employs only the standard displacement and traction boundary integral equations and can be formulated as a special case of residual method. Simplicity and stability can be mentioned as two appropriate characteristics of this formulation when applied to the solution of

\* Corresponding author.

E-mail address: [bomidvar@ut.ac.ir](mailto:bomidvar@ut.ac.ir) (B. Omidvar).