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Analysis of elastostatic problems using a semi-analytical method with diagonal coefficient matrices

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ABSTRACT

A new semi-analytical method is proposed for solving boundary value problems of two-dimensional elastic solids, in this paper. To this end, the boundary of the problem domain is discretized by specific non-isoparametric elements that are proposed for the first time in this research. These new elements employ higher-order Chebyshev mapping functions and new special shape functions. For these shape functions, Kronecker Delta property is satisfied for displacement function and its derivatives. Furthermore, the first derivatives of shape functions are assigned to zero at any given control point. Eventually, implementing a weak form of weighted residual method and using Clenshaw–Curtis quadrature, coefficient matrices of equations system become diagonal, which results in a set of decoupled governing equations to be used for solving the whole system. In other words, the governing equation for each degree of freedom (DOF) is independent from other DOFs of the problem. Validity and accuracy of the present method are fully demonstrated through four benchmark problems which are successfully modeled using a few numbers of DOFs. The numerical results agree very well with the analytical solutions and the results from other numerical methods.

1. Introduction

To solve elastostatic problems for analysis and design purposes, numerical approaches are usually employed. Different types of numerical approaches such as Finite Element Method (FEM), Boundary Element Method (BEM), Scaled Boundary Finite Element Method (SBFEM), and meshless methods are routinely used to solve elastostatic problems.

The use of FEM is advantageous as its procedures are well-established and versatile in nature (see for example [1,2]). On the other hand, BEM requires basically reduced surface discretizations, and may be considered as an appealing alternative to FEM for elastostatic problems ([3–12]). Since BEM does not require domain discretization, fewer unknowns are required to be stored. BEM needs a fundamental solution for the governing differential equation to obtain the boundary integral equation. Although coefficient matrices of BEM are much smaller than those of FEM, they are routinely non-positive definite, non-symmetric, and fully populated.

Combining the advantages of FEM and BEM, SBFEM was successfully developed [13]. Using surface finite elements, SBFEM discretizes only the boundary of the domain by transforming the governing partial differential equations to ordinary differential equations, which may be solved analytically. SBFEM, which requires no fundamental solution, have also been employed for the analysis of elastostatic problems (see for example [14–17]).

Researchers have paid attention to meshless methods during the last decade. These methods are the mesh reduction methods with no mesh requirements. Many meshless methods have been presented among which, various Petrov–Galerkin methods [18–20], element free Galerkin method [21,22], various boundary integral equation methods [23,24], different least-square methods [25,26], point interpolation method [27], hybrid methods [28,29], and other methods [30–33] have been employed in the analysis of elastostatic problems.

In this paper, two-dimensional (2D) elastostatic problems are studies using a novel semi-analytical method. The present research is the extension of the previous work of the authors [34], for solving potential problems. Therefore, the main concepts of the method are given in this paper. In other words, emphasis is mainly devoted to those important aspects of the method, as subjected to remarkable modifications compared to the previous work [34]. In this method, similar to the previous work for solving potential problems, only domain boundaries are discretized. Therefore, the governing partial differential equations are analytically solved in the problem domain. The elements of domain boundary are of special non-isoparametric ones. For these elements, new special shape functions and higher-order Chebyshev

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