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# A local radial basis functions—Finite differences technique for the analysis of composite plates

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ABSTRACT

Radial basis functions are a very accurate means of solving interpolation and partial differential equations problems. The global radial basis functions collocation technique produces ill-conditioning matrices when using multiquadrics, making the choice of the shape parameter a crucial issue. The use of local numerical schemes, such as finite differences produces much better conditioned matrices. However, finite difference schemes are limited to special grids. For scattered points, a combination of finite differences and radial basis functions would be a possible solution. In this paper, we use a higher-order shear deformation plate theory and a radial basis function—finite difference technique for predicting the static behavior of thin and thick composite plates. Through numerical experiments on square and L-shaped plates, the accuracy and efficiency of this collocation technique is demonstrated, and the numerical accuracy and convergence are thoughtfully examined. This technique shows great potential to solve large engineering problems without the issue of ill-conditioning.



## 1. Introduction

The analysis of laminated composite plates has been carried out by numerical techniques based on finite differences or finite elements. Historical view of the shear deformations theories can be found in [1–3]. More recently, meshless methods proved to be an interesting alternative as they avoid the meshing phase in the analysis. Two main meshless approaches have been used in the past: the weak-form and the strong-form techniques. Weak-form methods include for example the element-free Galerkin [4,5] or the Petrov–Galerkin [6,7] methods where a weight-residual approach is commonly used. The strong-form methods typically consider the interpolation directly on the equations of motion.

Radial basis functions were used by Hardy [8,9] for the interpolation of geographical scattered data and later used by Kansa [10,11] for the solution of partial differential equations (PDEs), with a global collocation. The global collocation proposed by Kansa considers a set of points distributed over a domain and boundary of the problem. Each point is connected, through a radial basis function, to all the remaining points of the nodal set. This global collocation generally produces dense, unsymmetrical, ill-conditioned matrices, which in turn can produce poor results and instability in the solution. However, when properly used, the global collocation produces excellent results. Also, using compact support functions with Kansa's method can produce sparse matrices at the cost of lower accuracy. Previously the authors successfully applied this meshless global collocation method to analyse plates and shells [12–23].

As a result of the ill-conditioning, some authors proposed a radial basis function method with a local approach [24–28]. The idea is to use radial basis functions with a local collocation as in finite differences, reducing the number of connections (the so-called support) for each node (also called center), hence producing a sparse matrix. This local approach retains many of the advantages of the global collocation, yet reducing the conditioning of the matrix. It is expected that the choice of the shape parameter will no longer be a critical issue, as in the case of global collocation.

In the present study we implemented, for the first time, the third-order theory of Reddy [29,30] with a local mesh-free method: the local radial basis function via finite difference approach method (RBF-FD), to model square isotropic, composite, sandwich and L-shaped isotropic plates. We compare the present local RBF-FD with exact solutions or other numerical techniques and the global Kansa collocations in some plate bending problems.

## 2. The RBF-FD method

The basic formulation of the RBF-FD method is presented. The finite difference method approximates derivatives of a function

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