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A hypersingular boundary integral analysis of axisymmetric steady-state heat conduction across a non-ideal interface between two dissimilar materials

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ABSTRACT

Steady-state axisymmetric heat conduction across a non-ideal interface between two dissimilar materials is considered. The non-ideal interface may be either low or high conducting. The relevant interfacial conditions are formulated in terms of hypersingular boundary integral equations. A simple boundary element procedure based on the hypersingular boundary integral formulations is proposed for solving numerically the axisymmetric heat conduction problem under consideration. Numerical results for some specific problems are obtained.

1. Introduction

If two dissimilar materials are bonded together with a very thin layer of material sandwiched in between them, the layer may be modeled as an interface in the form of a line (for plane problems) or a surface (in the case of three-dimensional problems). The boundary conditions to impose on the line or surface interface depend on the properties of the material in the thin layer and may be derived using asymptotic analysis (see, for example, [5]). Such a line or surface interface model helps to simplify the mathematics involved in the analysis of the multi-layered material.

In the context of heat conduction theory, the line or surface interface is regarded as thermally low conducting if the thin layer is occupied by a material of extremely low thermal conductivity. As an example, the interface between two imperfectly joined materials, which contains microscopic gaps filled with air, may be modeled as low conducting. On the other hand, if a material of extremely high thermal conductivity occupies the thin layer, the interface is said to be thermally high conducting. In a thermal system comprising a computer chip and a heat sink, the interface between the two components (the chip and the sink) may be modeled as high conducting if they are joined together by a thin layer of carbon nanotubes [7].

Interfacial conditions for thermally non-ideal interfaces that are either low or high conducting are given in Benveniste [4] and Miloh and Benveniste [8]. As one may intuitively expect, the temperature field varies continuously across a high conducting interface but it exhibits a jump across a low conducting interface. The normal heat flux is continuous on a low conducting interface but not on a high conducting one. The temperature jump across a low conducting interface is proportional in magnitude to the normal heat flux on the interface. For a high conducting interface, the jump in the normal heat flux is expressed in terms of second order spatial derivatives of the temperature on the interface.

To obtain a boundary element method for analyzing the two-dimensional steady-state temperature distribution in a bimaterial with a straight imperfect (low conducting) interface, Ang et al. [2] have derived a Green's function that satisfies appropriate conditions on the interface between two dissimilar half-spaces. A hypersingular boundary integral formulation is given by Ang [1] for two-dimensional heat conduction across an arbitrarily curved low conducting interface in a bimaterial.

The analysis in [1] is extended here to derive a hypersingular boundary integral formulation for axisymmetric steady-state heat conduction across a curved low conducting interface in a bimaterial. Moreover, the case of a high conducting interface is considered here. The extension is by no means trivial, as the fundamental solution of the axisymmetric heat equation is rather complicated being expressed in terms of the complete elliptic integrals of the first and second kind. Together with the boundary integral equation for axisymmetric heat conduction, the hypersingular boundary integral formulation for each of the two types of interfaces is used to derive a simple boundary element procedure for computing numerically the temperature distribution in the bimaterial. The boundary element procedure is applied to solve some particular problems involving axisymmetric heat

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