Engineering Analysis with Boundary Elements 35 (2011) 1116–1122

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Modeling domains using Bézier surfaces in plane boundary problems defined by the Navier–Lame equation with body forces

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ARTICLE INFO

Article history:
Received 18 October 2010
Accepted 18 April 2011
Available online 7 June 2011

ABSTRACT

The paper presents an effective and powerful way of solving boundary problems modeled by the Navier–Lame equation with body forces. We considered domains of complex shapes defined using Bézier surfaces, which are well-known in computer graphics. Obtained results were compared with analytical and numerical solutions received by the boundary element method (BEM). The analysis of the results has confirmed reliability and effectiveness of the proposed approach and its applicability to a variety of practical problems.

1. Introduction

With the advent of BEM [1,2], there appeared a method characterized by the reduction of the complexity of the problem by one order in relation to the most popular finite element method (FEM) [3]. The reduction comes from the fact that the method could be defined only on the boundary, which eliminates the need for discretization and allows to consider only the boundary. However, this valuable advantage of the new method turned out to be immaterial in the case of problems modeled by the Poisson equation or problems from the range of elasticity with body forces, because of the required calculation of integrals over the area. Then it was necessary to divide the considered area into so-called cells, which are sub-areas with more basic shapes. That procedure is very similar to the discretization using finite elements in FEM. In some cases (e.g. gravitational forces, centrifugal forces), it was possible to transform the area integral to the integral along the boundary [4]. For that reason, the method further preserved its benefits, but at the same time has ceased to be universal, since such a procedure is applicable only in selected situations.

Taking into account the above mentioned shortcomings of BEM, it is reasonable to look for an approach, which could eliminate them. Our previous research on solving boundary problems resulted in the creation of the parametric integral equation system (PIES) [5,6]. It is a modified version of the classic boundary integral equation (BIE) [1,7]. That modification consisted of the analytic inclusion of the boundary geometry, defined by appropriate curves, directly in the mathematical formalism of PIES. The strategy used resulted in an effective way of modeling the boundary with any curves of computer graphics, without discretization. In the case of polygons the number of used curves is the same as the number of sides [5,8], whilst domains with curved edge are modeled using as many curves as is required by an accurate definition of the considered shape [6,9].

PIES has been successfully used to solve various boundary value problems, starting from the simplest ones modeled by the Laplace equation [5,6,9], to the more complex ones of the acoustics [10] and linear elasticity without body forces [11,8]. For these problems the method has been thoroughly tested, and the obtained results have been compared with analytical and numerical results received using FEM and BEM [12].

Therefore it was decided to apply the approach to problems modeled by the Navier–Lame equation, restricted to the cases when the presence of body forces is not ignored. At first only problems defined on the polygonal domains were considered [13]. With such a strategy, a preliminary verification of the effectiveness and efficiency of the proposed approach was made. In all these cases, the mentioned approach gave accurate results with low complexity of modeling and numerical resolution.

The purpose of this study is to apply the developed approach to problems with more complex shapes, especially with the curved edge. This implies the need to apply Bézier patches of degree higher than the first to the area modeling and high order quadrature for integration over it. One of the major issues raised in this paper is the applicability of the standard quadrature for integration in the heavily modified areas and the effectiveness of the modeling of these areas using surface patches.

The paper presents several examples defined on areas of various shapes, exposed to various types of body forces. The results obtained were compared with analytical and numerical

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doi:10.1016/j.enganabound.2011.04.005