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Facial geometry parameterisation based on Partial Differential Equations

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ABSTRACT

Geometric modelling using Partial Differential Equations (PDEs) has been gradually recognised due to its smooth instinct, as well as the ability to generate a variety of geometric shapes by intuitively manipulating a relatively small set of PDE boundary curves. In this paper we explore and demonstrate the feasibility of the PDE method in facial geometry parameterisation. The geometry of a generic face is approximated by evaluating spectral solutions to a group of fourth order elliptic PDEs. Our PDE-based parameterisation scheme can produce and animate a high-resolution 3D face with a relatively small number of parameters. By taking advantage of parametric representation, the PDE method can use one fixed animation scheme to manipulate the facial geometry in varying Levels of Detail (LODs), without any further process.

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1. Introduction

The subject of Partial Differential Equations (PDEs) emerged in the 18th century as ordinary differential equations failed to describe some physical phenomena. Since then, many physical phenomena and paramount discoveries have been branded with the PDEs. Geometric modelling using PDEs has been widely studied in computer graphics since Bloor et al.'s PDE method was first introduced in blend surface generation two decades ago [1]. PDE methods adopt a boundary value approach whereby 3D geometric models can be reconstructed by solving PDEs either analytically or numerically with relevant boundary conditions. Advantages of the PDE method have been gradually recognised by researchers. A principal advantage comes from the ability that differential operators of the PDEs can ensure a generation of smooth surfaces, where the smoothness is strictly governed by the order of the PDEs used. A second advantage of using the PDE methods is that a PDE surface can be generated by intuitively manipulating a relatively small set of boundary curves. Moreover, the behaviour of PDE surfaces has been proven compatible with those underlying tensor-product surfaces, such as Bezier surface [2], *B*-splines [3], etc. Taken together, these advantages have contributed to a widespread adoption of the PDE methods in a range of disciplines, such as free-form surface design [4], solid modelling [5], computer aided manufacturing [6,7], shape morphing [8], web visualisation [9], mesh reconstruction [10], etc. In this paper, we address a prolonged topic, facial geometry parameterisation, by exploring and examining the feasibility of one PDE method in face modelling and facial animation.

Facial geometry parameterisation has been playing a crucial role in many fields, such as in the film industry and the games industry where 3D graphical models are used to enhance vividness and performance, in telecommunication where a 3D face is employed to improve coding efficiency for video conferencing [11], and even in medical operation where the 3D face model can be utilised to simulate plastic surgery. Efficient modelling of the human face and precise simulation of its expression

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