



Application of the MFS to inverse obstacle scattering problems

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ABSTRACT

In this paper, the method of fundamental solutions (MFS) is used to detect the shape, size and location of a scatterer embedded in a host acoustic homogeneous medium from scant measurements of the scattered acoustic pressure in the vicinity of the obstacle. A nonlinear constrained minimization regularized MFS technique is proposed for the numerical solution of the inverse problem in question. The stability of the technique is investigated by inverting measurements contaminated by random noise. The results of several numerical experiments are presented.

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1. Introduction

The inverse problems of time harmonic scattering of acoustic waves (that could be electromagnetic as well) from obstacles of arbitrary shape embedded in a fluid, or solid medium have been of considerable interest to researchers for many years, see e.g. [7,35] and references therein. In addition to being of academic interest, these problems have physical applications in the fields of radar and sonar detection; the ability to “see” in real time in complete darkness in murky water for deep sea submarines, underwater surveillance and target acquisition, detection of objects in the ocean, either fully submerged or partially buried in the seafloor, ultrasound medical imaging of soft tissues, nondestructive testing of materials, etc.

The inverse problem we consider in this paper is to determine the boundary of an insonified scatterer from scant measurements of its response when excited by impinging plane waves. The scatterer can be sound-soft, sound-hard or convectively embedded in the full-space. Although some uniqueness results are known, notably in [8], the problem is still difficult to solve since it is nonlinear, ill-posed (unstable), and computationally intensive in the high-frequency regime, see [30].

Integral equation methods, see e.g. [18,16], provide a basic tool in scattering theory mainly due to the fact that the formulation of obstacle scattering problems leads to boundary value problems defined over unbounded domains. Hence, their formulation in terms of boundary integral equations not only reduces by one the dimensionality of the problem, but also replaces a problem over an unbounded domain by one over a bounded contour, or surface. From a numerical point of view both these advantages have made

integral equation methods, such as the boundary element method (BEM), some of the most powerful techniques for the approximate solution of obstacle scattering in homogeneous media, see [29].

The method of fundamental solutions (MFS) may be viewed as a meshless BEM in which the governing field is approximated by a linear combination of fundamental solutions for the Helmholtz equation whose singularities are placed at discrete points or on a continuous curve outside the solution domain, see [31]. The version of the MFS in which these fictitious singularities (internal sources) are fixed, is also called the charge simulation technique or the method of internal sources, see [38]. In contrast to the BEM, the MFS provides accurate approximations directly at points arbitrarily close to the boundary. Moreover, since no shape functions or elements are involved, it is considerably easier to implement than the BEM. Also, the fictitious eigenfrequency difficulty which occurs with the BEM for the exterior Helmholtz problem apparently is not present in the MFS, see [27].

Up to now, the MFS has never been applied to inverse obstacle scattering, although its principal idea of representing the solution as a single-layer potential has been used previously by Kirsch and Kress [25], and Kress and Zinn [32]. For other recent applications and development of the MFS for shape identification problems, see [2,3,6,20–22,33].

In this paper, the inverse, nonlinear and ill-posed problem of determining the boundary of the obstacle from Cauchy data ultrasound boundary measurements in acoustic scattering is approached based on a regularized optimization procedure which uses the MFS solver at each iteration until a prescribed stopping criterion is satisfied. Some preliminary results on this approach have been reported in [23]. A similar approach using the BEM instead of MFS has been previously investigated in [34], but only for sound-hard obstacles parameterised by a maximum of seven parameters and for exact scattered wave data only.

The method proposed in this paper can be easily extended to penetrable obstacles, as well as to electromagnetic scattering problems.

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