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Fast identification of cracks using higher-order topological sensitivity for 2-D potential problems

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This article concerns an extension of the topological sensitivity (TS) concept for 2D potential problems involving insulated cracks, whereby a misfit functional J is expanded in powers of the characteristic size a of a crack. Going beyond the standard TS, which evaluates (in the present context) the leading $O(a^2)$ approximation of J, the higher-order TS established here for a small crack of arbitrarily given location and shape embedded in a 2-D region of arbitrary shape and conductivity yields the $O(a^4)$ approximation of J. Simpler and more explicit versions of this formulation are obtained for a centrally symmetric crack and a straight crack. A simple approximate global procedure for crack identification, based on minimizing the $O(a^4)$ expansion of J over a dense search grid, is proposed and demonstrated on a synthetic numerical example. BIE formulations are prominently used in both the mathematical treatment leading to the $O(a^4)$ approximation of J and the subsequent numerical experiments.

1. Introduction

The sensitivity analysis of objective functions has a firm mathematical basis and provides efficient computational tools for e.g. optimal design or inversion of experimental data. Along with classical sensitivity methods, which perform first-order perturbation analyses with respect to small variations of some feature of the system under consideration, another sensitivity concept, namely that of topological sensitivity, appeared more recently in the context of topological optimization of mechanical structures [14,26]. Topological sensitivity is concerned with quantifying the perturbation of an objective function J with respect to the nucleation of a small object $D_a(\boldsymbol{z})$ of characteristic linear size a and specified location $\boldsymbol{z}$, as a function of $\boldsymbol{z}$. Denoting by $J(a;\boldsymbol{z})$ the value achieved by J when $D_a(\boldsymbol{z})$ is the only perturbation to an otherwise known reference medium, then in 2-D situations with Neumann or transmission conditions on $\partial D_a(\boldsymbol{z})$ the topological derivative $\mathcal{T}_2(\boldsymbol{z})$ appears through an expansion of the form

$$J(a;\boldsymbol{z}) = J(0) + a^2\mathcal{T}_2(\boldsymbol{z}) + o(a^2).$$

Subsequent investigations have also established the usefulness of the topological sensitivity as a preliminary sampling tool for defect identification problems, providing estimates of location, size and number of a set of sought defects [6,10,15,18,19,21,22]. This approach has also been extended by formulating higher-order expansions of $J(a;\boldsymbol{z})$. The $O(a^4)$ expansion of cost functionals involving the solution of a 2-D potential problem in a domain of arbitrary shape containing a small inclusion of size a, of the form

$$\begin{aligned} J(a;\boldsymbol{z}) &= J(0) + \mathcal{T}_2(\boldsymbol{z})a^2 + \mathcal{T}_3(\boldsymbol{z})a^3 + \mathcal{T}_4(\boldsymbol{z})a^4 + o(a^4) \\ &\equiv J(0) + J_4(a;\boldsymbol{z}) + o(a^4), \end{aligned} \tag{1}$$

where coefficients $\mathcal{T}_2,\mathcal{T}_3,\mathcal{T}_4$ depend on the assumed characteristics of the small nucleating object, is established in [9] (for arbitrary cost functionals expressed as boundary integrals and inclusions of arbitrary shape) and [25] (for potential energy and circular inclusions), while similar expansions for the scattering by sound-hard obstacles are established in [8] for problems governed by the 3-D Helmholtz equation. The concept of topological sensitivity, and higher-order topological expansions such as (1), are particular instances of the broader class of asymptotic methods, where approximate solutions to problems featuring a small geometrical parameter a are sought in the form of expansions with respect to a, see e.g. [2,23].

This article is a continuation of [9] where the small nucleating object is a perfectly insulating crack, through which discontinuities of the potential are allowed. Its aim is twofold: (i) to establish the expressions of coefficients $\mathcal{T}_2,\mathcal{T}_3,\mathcal{T}_4$ for a crack of size a embedded in a 2-D medium characterized by a scalar conductivity, permitting computationally efficient methods for evaluating small-crack expansions of cost functionals, and (ii) to demonstrate the effectiveness of the resulting expansion (1) for crack identification purposes. Since the sensitivity of cost functionals (rather than field variables) is emphasized here, an adjoint solution-based approach is formulated as it avoids the

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