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Computation of moments and stresses in laminated composite plates by the boundary element method

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ABSTRACT

This paper presents a boundary element formulation for the computation of moments and stresses at internal and boundary points of laminated composite plates. Integral equations for second transversal displacement derivatives are developed and all derivatives of the fundamental solution are computed analytically. These integral equations are used to compute moments and stresses at internal points. Stresses on the boundary are computed by a procedure that uses integral equations for the first transversal displacement derivatives, derivatives of shape functions, and constitutive relations. The obtained results are in good agreement with finite element results available in literature.

1. Introduction

The attempt of developing analytical models for the representation of the behavior of plates comes since middle of 1800 with works developed by Sophie Germain, Lagrange, and Poisson [1]. Since 1978, when the first general direct formulation based on Kirchhoff's hypothesis appeared, the boundary element method (BEM) has had large growth, being nowadays applied to several practical engineering problems. The first works discussing the use of boundary element direct formulation, in conjunction with Kirchhoff's theory, were by Bezine [2], Stern [3], and Tottenhan [4]. Nowadays, BEM is a well-established numerical technique to deal with an enormous number of engineering complex problems. Analysis of plate bending problems using the BEM has attracted the attention of many researchers during the past years, proving to be a particularly adequate field of applications for that technique. The fundamental solution is an essential part of the boundary element method. Bending analysis of thin plates by the BEM requires the use of two fundamental solutions: the displacement field due to a transverse point load, and the displacement field due to a unit point moment. Fundamental solutions for anisotropic plates utilize complex variable theory following Lekhnitskii [5]. Shi and Bezine [6] presented a boundary element analysis of plate bending problems using fundamental solutions proposed by [7] based on Kirchhoff plate bending assumptions. Rajamohan and Raamachandran [8] proposed a formulation where singularities were avoided by placing source points outside the domain. Paiva et al. [9] presented an analytical treatment for singular and hypersingular integrals of the formulation presented in [6]. Albuquerque et al. [10] presented a boundary only integral formulation for the classical plate theory of composite laminate materials. The transformation of domain integrals into boundary integrals follows the radial integration method, as proposed by Gao [11]. In [12], this formulation was extended for dynamic problems. Shear deformable shells have been analyzed using the boundary element method by [13] with the analytical fundamental solution proposed by [14]. Wang and Huang [15] presented a boundary element formulation for orthotropic shear deformable plates. Later, in Wang and Schweizerhof [16], the previous formulation was extended to laminate composite plates. Recently, Wen et al. [17] presented a displacement discontinuity formulation for modeling cracks in orthotropic Reissner plates.

Stress and moment computation by the BEM has been addressed by some works in literature. For example, Zao [18] and Zao and Lan [19] have discussed the computation of stresses in plane elastic problems, Knopke [20] presented and discussed the integral formulation for computation of stresses in isotropic thin plate, Rashed et al. [21] presented an stress integral formulation in the BEM for Reissner plate bending problems. To the best of authors knowledge, the computation of stresses by the BEM in anisotropic plates have still not been addressed in literature.

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