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RBFs approximation method for Kawahara equation

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ABSTRACT

In this paper, we apply radial basis function (RBF) method for the numerical solution of Kawahara equation. Derivatives of interpolation are used to approximate the spatial derivatives and the temporal derivative approximated by a low order forward difference scheme. Computational experiments are performed for single soliton, two solitons, and three solitons interaction to examine accuracy of the method. The results obtained by the present method are in good agreement with exact solutions and some earlier work in literature. Accuracy of the method is estimated in terms of the error norms L_2, L_∞, the three invariants, number of nodes in the domain of influence, time step size, parameter dependent, and parameter independent RBFs.

1. Introduction

The shallow water waves in a dispersive system can be described by the third order Kortewege–de Vries (KdV) equation [1,2]. The fifth order KdV equation is a mathematical model for shallow water waves having surface tension and acoustic waves in plasma [3,4]. Unlike the third order KdV equation, the fifth order KdV equation is non-integrable [5]. Although the most general solution of fifth order Kdv equation is not available, the analytical solution for a special case in the form of solitary waves is given in [6]. Very few methods have been applied for the numerical solution of fifth order KdV equation [3,7,8].

In this paper, we propose RBFs interpolation method for the numerical solution of the Kawahara equation [8]:

$$\frac{\partial Y}{\partial t}+\left(\frac{105}{16}\right)\alpha^2 Y\frac{\partial Y}{\partial x}+\left(\frac{13}{4}\right)\beta\frac{\partial^3 Y}{\partial x^3}-\frac{\partial^5 Y}{\partial x^5}=0,\quad a\le x\le b,\ t>0. \tag{1}$$

Modified multiquadric (MQ) scheme was introduced by Kansa [10] in 1990 to find out numerical solution of partial differential equations. The existence, uniqueness, and convergence of the method were studied by Micchelli [11], Madych [12] and Frank and Schaback [13]. Solvability of the system obtained in MQ method was studied by Micchelli in 1986 for distinct interpolation points. Hon and Mao [14] extended the use of MQ to numerical solutions of various ordinary and partial differential equations including nonlinear Burgers equation with shock waves. Various researchers have contributed recently to this field (see Refs. [15–22], etc.)

The structure of the present paper is organized as follows: In Section 2 the method is discussed, in Section 3 the numerical test of the method on the problems related to the Kawahara equation are given while in Section 4 the results are concluded.

2. Analysis of the technique

In this section, an r-dimensional ($r=1,2,3$) time dependent boundary value problem given by

$$\frac{\partial Y}{\partial t}+\mathcal{L}Y=g(x,t),\ x\in\Gamma,\quad \mathcal{B}Y=h(x,t),\ x\in\partial\Gamma, \tag{2}$$

supplemented by initial condition

$$Y(x,0)=Y_0(x),$$

where $\mathcal{L}$, $\mathcal{B}$ are differential and boundary operators and Γ and $\partial\Gamma$ represent inner and boundary regions of the domain, respectively.

Applying θ-weighted scheme to Eq. (2) in the form

$$\frac{Y^{n+1}-Y^n}{\delta t}+\theta\mathcal{L}Y^{n+1}+(1-\theta)\mathcal{L}Y^n=g(x,t^{n+1}),\quad 0\le\theta\le 1, \tag{3}$$

where δt denotes time step and Y^n (n is non-negative integer) indicates solution at time $t^n=n\delta t$. Let $\{x_i\}_{i=1}^{K}$ be collocation points in the domain, and among them, $\{x_i\}_{i=1}^{K_b}$ interior and $\{x_i\}_{i=K_b+1}^{K}$ boundary points, respectively. We approximate the

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