Perturbation technique and method of fundamental solution to solve nonlinear Poisson problems

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ABSTRACT

We show in this work that the Asymptotic Numerical Method (ANM) combined with the Method of Fundamental Solution (MFS) can be a robust algorithm to solve the nonlinear Poisson problem. The ANM transforms the nonlinear problem into a sequence of linear ones which can be solved by MFS. This last method consists of approximating the solution of the linear Poisson problem by a linear combination of fundamental solutions. Some examples are presented to show the efficiency of the proposed method.

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1. Introduction

A few decades ago we have shown the efficiency of the ANM to compute the solution of nonlinear partial differential equations. Many applications have established the robustness of this method for nonlinear problems in solid and fluid mechanics, nonlinear vibrations, contact, large displacement and rotations, plasticity and other fields in physics [1–14].

ANM consists in computing the solution into power series with respect to a scalar parameter. This allows one to transform the nonlinear problem into a sequence of linear ones which have the same tangent operator. Consequently only one tangent matrix triangulation is needed to compute all the terms of the series. As the series have a limited convergence radius, the technique of Padé approximants is used to improve the validity range of the solution [7,8]. Up to now, ANM is generally associated to Finite Element Method to solve the resulting linear problems.

In recent years, there have been increasing interests in using meshfree techniques which aim to avoid the meshing restrictions encountered in the classical Finite Element Method. Several techniques have been proposed, but here, we are particularly interested in the so called Method of Fundamental Solutions (MFS) for the simplicity of its numerical implementation.

The main idea of this method consists of approximating the solution of the problem by a linear combination of fundamental solutions with respect to some source points which are located outside the domain. Then, the original problem is reduced to determining the unknown coefficients of the fundamental solutions by requiring the approximation to satisfy the boundary conditions.

MFS was first proposed by Kupradze and Aleksidze [15] and has been applied to many physical problems represented by linear differential equations, such as Laplace equation, Poisson's equation, eigenvalue problem, Helmholtz equation, Stokes equations, inverse problems, plate bending problems, etc. [16–26]. This method has been extended to solve some nonlinear problems [27-32]. It was mainly combined with iterative methods as Newton–Raphson method, Picard iteration [19,20] or concept of a matrix particular solution [22]. The association of MFS and ELM (Eulerian–Lagragian method) has been used to deal with nonlinear problems successfully, such as advection–diffusion equations [28], Burgers' equation and Navier–Stokes equations [29,30]. The Trefftz method and MFS have been intensively investigated by Balakrishnan and Ramachandran [22,23] for nonlinear problems in heat and mass transfer.

Many methods have been proposed in the last decade to improve MFS. In these papers, a key point is the introduction of various shape functions to discretize the considered linear problems. The Analog Equation Method (AEM) [21,31,40] allows solving linear equations even if a fundamental solution is not known, what is of high interest when dealing with nonlinear equations. The idea is to use shape functions that are solutions of another "analog" equation, but it is no longer a boundary-only technique as MFS. There are other interesting principles to create other shape functions, for instance by starting from Helmholtz operator instead of Laplacian or by finding a family of linear operators whose combination cancels the right hand side: this has led to the boundary knot method [34] and the boundary

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