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# Mixed types of boundary conditions at corners of linear elastostatics and their numerical solutions

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ABSTRACT

The singular solutions at corners and the fundamental solution are *essential* in both theory and computation. Our recent efforts are made to seek the particular solutions of corner and crack singularity of linear elastostatics, to design new models of corner singularity, and to find their numerical solutions. In [1,2], a systematic analysis for singularity properties and particular solutions of linear elastostatics is explored, and the singular solutions for corners with the displacement or the free traction boundary conditions have been found. This paper is a continued study of [1,2], to explore new particular solutions for mixed types of boundary conditions at corners, which mean that the displacement and the free traction boundary conditions are subjected to the same corner edge in this paper. Explicit particular solutions have been found for any angle $\Theta \in (0,2\pi]$; this is *distinct* from [1,2] where the explicit solutions only with $\Theta = \pi$ and $\Theta = 2\pi$ can be obtained. In this paper new singularity models with L-shaped domain and other non-rectangular domains are designed, and the highly accurate solutions are computed. Moreover, the singularity solutions as $O(r^{1/4})$ and even $O(r^{1/7})$ are found (Refs. [1–3]). To our best knowledge, this is the first time to provide the particular solutions with different boundary conditions on the same corner edge in linear elastostatics. The new particular solutions, new singularity, analysis, and computation in this paper are important for both theory and computation of linear elastostatics.



## 1. Introduction

The singular solutions at corners and the fundamental solution are *essential* in both theory and computation. Our recent efforts are made to seek the particular solutions of corner and crack singularity of linear elastostatics, to design new models of corner singularity, and to find their numerical solutions. In [1,2], a systematic analysis for singularity properties and particular solutions of linear elastostatics is explored, and the singular solutions for corners with the displacement or the free traction boundary conditions have been found. This paper is a continued study of [1,2], to explore new particular solutions for mixed types of boundary conditions at corners, which mean that the displacement and the free traction boundary conditions are subjected to the same corner edge in this paper. Explicit particular solutions have been found for any angle $\Theta \in (0,2\pi]$; this is *distinct* from [1,2] where the explicit solutions only with $\Theta = \pi$ and $\Theta = 2\pi$ can be obtained. In this paper new singularity models with L-shaped domain and other non-rectangular domains are designed, and the highly accurate solutions are computed. Moreover, the singularity solutions as $O(r^{1/4})$ and even $O(r^{1/7})$ are found (Refs. [1–3]). To our best knowledge, this is the first time to provide the particular solutions with different boundary conditions on the same corner edge in linear elastostatics. The new particular solutions, new singularity, analysis, and computation in this paper are important for both theory and computation of linear elastostatics. The singular properties for traction conditions in Williams [4,5] are obtained by using a similarity to biharmonic equations, and some discussions and applications are given in Lin and Tong [5], Jirousek and Venkstesh [6], Jirousek and Wroblewski [7] and Qin [8,2]. In [1,2] and this paper, we will derive the particular solutions directly from the Cauchy–Navier equation of linear elastostatics.

Once the singularity of corner solutions is known, the reduced convergence rates of FEM, FDM and FVM are found, and some improved techniques, such as the combined Trefftz method [10], can be explored to recover the optimal convergence rates. More importantly, based on the explicit particular solutions of corners given in [1,2] and this paper, we may develop a number of

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