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Two variational formulations for elastic domain decomposition problems solved by SGBEM enforcing coupling conditions in a weak form

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ABSTRACT

The solution of Boundary Value Problems of linear elasticity using a domain decomposition approach (DDBVPs) is considered. Some theoretical aspects of two new energy functionals, adequate for a formulation of symmetric Galerkin boundary element method (SGBEM) applied to DDBVPs with non-conforming meshes along interfaces, are studied. Considering two subdomains Ω^A and Ω^B, the first functional, $E(\boldsymbol{u}^A,\boldsymbol{u}^B)$, is expressed in terms of subdomain displacement fields, and the second one, $\Pi(\boldsymbol{u}^A,\boldsymbol{u}^B,\boldsymbol{t}^A,\boldsymbol{t}^B)$, in terms of unknown displacements and tractions defined on subdomain boundaries. These functionals generalize the energy functionals studied in the framework of the single domain SGBEM, respectively, by Bonnet [Eng Anal Boundary Elem 1995;15:93–102] and Polizzotto [Eng Anal Boundary Elem 1991;8:89–93]. First, it is shown that the solution of a DDBVP represents the saddle point of the functional E. Second, it is shown that the solution of an SGBEM system of boundary integral equations for a DDBVP corresponds to the saddle point of the functional Π. Then, the functional Π is considered for the finite-dimensional spaces of discretized boundary displacements and tractions showing that the solution of the SGBEM linear system of equations represents the saddle point of Π, generalizing in this way the boundary min–max principle, introduced by Polizzotto, to SGBEM solutions of DDBVPs. Finally, a relation between both energy functionals is deduced.

1. Introduction

The development of numerical techniques for solution of boundary value problems (BVP) *via* domain decomposition (DDBVPs) has substantially increased in the last decade. There exist several approaches to the mathematical formulation and solution of DDBVPs, see e.g. the work of Mathew [1], Quarteroni and Valli [2]. Different formulations of the boundary element method (BEM) [3] in its symmetric Galerkin form, known as SGBEM, see Bonnet et al. [4] and Sutradhar et al. [5], applied to DDBVPs with conforming interface meshes can be found in Ganguly et al. [6], Gray and Paulino [7], Hsiao et al. [8], Kallivokas et al. [9], Maier et al. [10], Mantič [11] and Panzeca et al. [12]. A variational principle with Lagrange multipliers for the coupling of FEM and collocational BEM when solving elasto-static and elasto-dynamic problems with non-conforming interface meshes was presented by Rüberg and Schanz [13].

A novel approach to an application of SGBEM for DDBVPs has recently been introduced in Vodička et al. [14], generalizing the single-domain SGBEM variational formulation by Bonnet [15]. This approach is based on an energy functional for DDBVPs proposed in [14], which results in a new variational formulation of the SGBEM for DDBVPs with a weak imposition of coupling conditions, thus allowing non-conforming meshes along interfaces between subdomains. It should be mentioned that another variational principle for solving thermal and acoustic DDBVPs by SGBEM was presented in Kallivokas et al. [9].

The SGBEM code developed in [14] has successfully been tested, evaluating accuracy and asymptotic convergence of the numerical results obtained for examples including non-matching meshes of linear continuous boundary elements [3] along straight and curved interfaces.

The theoretical base of this new variational formulation is, nevertheless, somewhat wider than was explained in the original work by Vodička et al. [14]. In view of this, a few new relevant theoretical results associated to the energy-based formulations for DDBVPs are introduced in the present paper.

First, after a concise formulation of an elastic DDBVP in Section 2, it is proved in Section 3 that the critical (stationary) point of the energy functional from [14] is indeed a saddle point.

Then, another variational principle, expressed only in terms of boundary and interface unknowns, is presented in Section 4. An analogous principle was introduced in Polizzotto [16] for

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