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# Splitting extrapolation algorithms for solving the boundary integral equations of Steklov problems on polygons by mechanical quadrature methods ☆

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ABSTRACT

By the potential theory, Steklov eigenvalue problems are converted into boundary integral equations (BIEs). The singularities at corners and in the integral kernels are studied in this paper. Mechanical quadrature methods (MQMs) are presented to obtain approximations with a high accuracy order $O(h^3)$. Moreover, the mechanical quadrature methods are simple without any singularly integral computation. Since the asymptotic expansions of the errors with the power $O(h^3)$ are shown, the high accuracy order $O(h^5)$ can be achieved for the solutions by using the splitting extrapolation algorithms (SEAs). A posteriori error estimate can also be obtained for self-adaptive algorithms. The efficiency of the algorithms is illustrated by examples.



## 1. Introduction

The Steklov eigenvalue problems are defined as follows:

$$\begin{cases} \Delta u = 0 & \text{in } \Omega, \\ \dfrac{\partial u}{\partial n} = \lambda u & \text{on } \Gamma, \end{cases} \tag{1}$$

where $\Omega \subset R^2$ is a bounded, simply connected domain with a piecewise smooth boundary $\Gamma$, $\Gamma(=\bigcup_{m=1}^{d}\Gamma_m)$ is a closed curve, $\partial/(\partial n)$ is an outward normal derivative on $\Gamma$, and $\lambda$ is an eigenvalue.

The problem arises from many applications, e.g. the free membranes and heat flow problems, the analysis of stability of mechanical oscillators, and the study of vibration modes of a structure interaction. Andreev and Todorov [2], Armentano and Padra [3] and Hadjesfandiari and Dargush [9] studied finite element methods and carried out the error estimation. Liu and Ortiz [15] provided finite difference methods and Tao-methods. Tang et al. [22] derived boundary element methods for smooth boundary $\Gamma$ and the accuracy orders of their approximation are $O(h^2)$. Huang and Lü [11] constructed the mechanical quadrature methods (MQMs) with the accuracy orders $O(h^3)$ for smooth boundary $\Gamma$.

By means of the potential theory, Eq. (1) will be transformed into general eigenvalue problems of boundary integral equations (BIEs) as follows (see [4,9,11]):

$$\alpha_i(y)u(y) - \int_\Gamma k^*(y,x)u(x)\,ds_x = \int_\Gamma h^*(y,x)\frac{\partial u(x)}{\partial n_x}\,ds_x, \quad y \in \Gamma_i, \tag{2}$$

where $\alpha_i(y) = \theta(y)/(2\pi)$ is related to the interior angle $\theta(y)$ of $\Omega$ at $y \in \Gamma_i$, especially when $y$ is on a smooth part of the boundary $\Gamma$, $\alpha_i(y) = 1/2$, $h^*(y,x) = -1/(2\pi)\ln r$ is the fundamental solution and $k^*(y,x) = \partial h^*(y,x)/\partial n$ with $r = |x-y|$.

The right hand side term of Eq. (2) is characterized by a logarithm singularity, and $\partial u/\partial n$ is discontinuous with singularity at the corners. There exist some difficulties to deal with the singularity and the discontinuity of the boundary integral equations on polygon (see [6]). To overcome such difficulties, various numerical methods have been proposed, such as Galerkin methods in Stephan and Wendland [21], Chandler [5], Sloan and Spence [20] and Amini and Nixon [1], the collocation methods in Elschner and Graham [8] and Yan [23], the quadrature methods in Sidi and Israrli [19], Saranen [17], Saranen and Sloan [18] and the combined Trefftz methods in Li [13]. Moreover, the mechanical quadrature methods have been developed by Huang and Lü [11,16], which have the convergence rate $O(h^3)$ and excellent stability. The MQMs are applied in many boundary integral equations (see [10–12,16,25]).

We present the mechanical quadrature methods (MQMs) of BIEs for solving Steklov eigenvalue problems by Side's quadrature rules [11,19], in which the generation of the discrete matrices does not require any calculations of singular integrals. After the asymptotic expansions of the errors are proved, a higher accuracy

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