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Strong convergence theorems for fixed point problems, variational inequality problems and system of generalized mixed equilibrium problems

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ABSTRACT

The purpose of this paper is to construct a new iterative scheme by hybrid methods to approximate a common element of the common fixed points set of a finite family of ϕ-asymptotically nonexpansive mappings, the solutions set of a variational inequality problem and the solutions set of a system of generalized mixed equilibrium problems in a 2-uniformly convex real Banach space which is also uniformly smooth. Then, we prove strong convergence of the scheme to a common element of the three sets. Our results extend many known recent results in the literature.

1. Introduction

Let E be a real Banach space with dual E^* and C be nonempty, closed and convex subset of E. A mapping $T : C \to C$ is called *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall x, y \in C. \tag{1.1}$$

A point $x \in C$ is called *a fixed point* of T if $Tx = x$. The set of fixed points of T is denoted by $F(T) := \{x \in C : Tx = x\}$.

We denote by J, the normalized duality mapping from E to 2^{E^*} defined by

$$J(x) = \{f \in E^* : \langle x, f\rangle = \|x\|^2 = \|f\|^2\}.$$

The following properties of J are well known (the reader can consult [1–3] for more details).

(1) If E is uniformly smooth, then J is norm-to-norm uniformly continuous on each bounded subset of E.
(2) $J(x) \neq \emptyset,\ x \in E$.
(3) If E is reflexive, then J is a mapping from E onto E^*.
(4) If E is smooth, then J is single valued.

Throughout this paper, we denote by ϕ, the functional on $E \times E$ defined by

$$\phi(x, y) = \|x\|^2 - 2\langle x, J(y)\rangle + \|y\|^2, \quad \forall x, y \in E. \tag{1.2}$$

Let C be a nonempty subset of E and let T be a mapping from C onto E. A point $p \in C$ is said to be an *asymptotic fixed point* of T if C contains a sequence $\{x_n\}_{n=0}^{\infty}$ which converges weakly to p and $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$. The set of asymptotic fixed points of T is denoted by $\widehat{F}(T)$. We say that a mapping T is *relatively nonexpansive* (see, for example, [4–8]) if the following conditions are satisfied:

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