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On the dynamics of the recursive sequence $x_{n+1} = \frac{\alpha x_{n-1}}{\beta+\gamma \sum_{k=1}^{t} x_{n-2k} \prod_{k=1}^{t} x_{n-2k}}$

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ABSTRACT

In this paper, we investigate the global behavior of the difference equation

$$x_{n+1} = \frac{\alpha x_{n-1}}{\beta + \gamma \sum_{k=1}^{t} x_{n-2k} \prod_{k=1}^{t} x_{n-2k}}, \quad n = 0, 1, \ldots$$

where β is a positive parameter and α, γ are non-negative parameters, with non-negative initial conditions.

1. Introduction

Consider the higher-order difference equation

$$x_{n+1} = \frac{\alpha x_{n-1}}{\beta + \gamma \sum_{k=1}^{t} x_{n-2k} \prod_{k=1}^{t} x_{n-2k}}, \quad n = 0, 1, \ldots \tag{1.1}$$

where the parameters, β is positive and α, γ are non-negative real numbers and the initial conditions $x_{-2t}, \ldots, x_{-2}, x_{-1}$ and x_0 are non-negative real numbers such that

$$0 < \beta + \gamma \sum_{k=1}^{t} x_{n-2k} \prod_{k=1}^{t} x_{n-2k}, \quad n = 0, 1, \ldots.$$

If $\alpha = 0$ the equation $x_{n+1} = 0$ is trivial, if $\gamma = 0$ the equation $x_{n+1} = \frac{\alpha}{\beta} x_{n-1}$ is linear, we assume that all parameters in equations are positive.

We investigate the global asymptotic behavior and the periodic character of the solutions of the difference equation (1.1), by generalizing the results due to El-Owaidy et al. [1] corresponding to the difference equation

$$x_{n+1} = \frac{\alpha x_{n-1}}{\beta + \gamma x_{n-2}^{p}}, \quad n = 0, 1, \ldots$$

where the parameters α, β and γ are positive real numbers and the initial conditions x_{-2}, x_{-1} and x_0 are arbitrary non-negative real numbers. Similar recursive sequences are studied previously; for example, see Refs. [1–22].

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