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Multiple integral representations for some families of hypergeometric and other polynomials

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ABSTRACT

The main objective of this paper is to investigate several general families of hypergeometric and other polynomials and their associated multiple integral representations. Each of the integral representations, which are derived in this paper, may be viewed also as a linearization relationship for the product of two different members of the associated family of hypergeometric and other polynomials.

1. Introduction and definitions

Let $\{A_{m,n}\}_{m,n=0}^{\infty}$ be a suitably bounded double sequence of essentially arbitrary (real or complex) parameters. Over three decades ago, Srivastava [1] considered the following general family of polynomials:

$$S_n^N(z) := \sum_{k=0}^{[n/N]} \frac{(-n)_{Nk}}{k!} A_{n,k} z^k \quad (n \in \mathbb{N}_0;\ N \in \mathbb{N}), \tag{1}$$

where $[\kappa]$ denotes the greatest integer in $\kappa \in \mathbb{R}$ and $(\lambda)_\nu$ denotes the Pochhammer symbol defined, in terms of the familiar Gamma function, by

$$(\lambda)_\nu := \frac{\Gamma(\lambda+\nu)}{\Gamma(\lambda)} = \begin{cases} 1 & (\nu = 0;\ \lambda \in \mathbb{C} \setminus \{0\}) \\ \lambda(\lambda+1)\cdots(\lambda+n-1) & (\nu = n \in \mathbb{N};\ \lambda \in \mathbb{C}), \end{cases}$$

$\mathbb{N}$ and $\mathbb{C}$ being, as usual, the set of *positive* integers and the set of *complex* numbers, respectively. It is also understood *conventionally* that $(0)_0 := 1$.

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