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An iterative method for fixed point problems, variational inclusions and generalized equilibrium problems

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ABSTRACT

In this paper, we introduce a new iterative scheme for finding a common element of the set of fixed points of a nonexpansive mapping, the set of solutions to a variational inclusion and the set of solutions to a generalized equilibrium problem in a real Hilbert space. Furthermore, using our new iterative scheme, we prove strong convergence theorems which extend many recent important results. Finally, we apply our results to solve an optimization problem in a real Hilbert space.

1. Introduction

Let H be a real Hilbert space with norm $\|.\|$ and inner product $\langle ., .\rangle$ and let K be a nonempty, closed and convex subset of H. A mapping $T : K \to K$ is called *nonexpansive* if

$$\|Tx - Ty\| \le \|x - y\|, \quad \forall x, y \in K. \tag{1.1}$$

A point $x \in K$ is called *a fixed point* of T if $Tx = x$. The set of fixed points of T is the set $F(T) := \{x \in K : Tx = x\}$.

A mapping $\psi : K \to H$ is called *inverse-strongly monotone* (see, for example, [1,2]) if there exists a positive real number μ such that $\langle \psi x - \psi y, x - y\rangle \ge \alpha\|\psi x - \psi y\|^2$, $\forall x, y \in K$. For such a case, ψ is called μ-inverse-strongly monotone.

Let $\psi : K \to H$ be a nonlinear mapping. The variational inequality problem (see, for example, [3,4]) is to find an $x \in K$ such that

$$\langle \psi x, y - x\rangle \ge 0, \quad \forall y \in K. \tag{1.2}$$

The set of solutions to the variational inequality problem (1.2) is denoted by $VI(K, \psi)$. Finding a common element of the set of fixed points of nonexpansive mappings and the set of solutions to the variational inequality problem has been studied extensively in the literature (see, for example, [5–12] and the references therein).

Let $A : H \to H$ be a single-valued nonlinear mapping and let $M : H \to 2^H$ be a set-valued mapping. The variational inclusion is to find $u \in H$ such that

$$\theta \in A(u) + M(u), \tag{1.3}$$

where θ is a zero vector in H. The set of solutions to the variational inclusion (1.3) is denoted by $I(A, M)$. When $A = 0$, then (1.3) becomes the inclusion problem introduced by Rockafellar [13].

A set-valued mapping $M : H \to 2^H$ is called *monotone* if for all $x, y \in H$, $f \in M(x)$ and $g \in M(y)$ imply $\langle x-y, f-g\rangle \ge 0$. A monotone mapping M is said to be *maximal* if the graph $G(M)$ is not properly contained in the graph of any other monotone map, where $G(M) := \{(x, y) \in H \times H : y \in Mx\}$ for a multi-valued mapping M. It is also known that M is maximal if and only if for $(x, f) \in H \times H$, $\langle x - y, f - g\rangle \ge 0$ for every $(y, g) \in G(M)$ implies $f \in Mx$. The resolvent operator $J_{M,\lambda}$ associated

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