Mathematical and Computer Modelling 54 (2011) 1365–1379

Contents lists available at ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Error bounds for weighted 2-point and 3-point Radau and Lobatto quadrature rules for functions of bounded variation

A. Aglić Aljinović [a,*], S. Kovač [b], J. Pečarić [c]

[a] Department of Applied Mathematics, Faculty of Electrical Engineering and Computing, University of Zagreb, Unska 3, 10000 Zagreb, Croatia
[b] Faculty of Geotechnical Engineering, University of Zagreb, Hallerova aleja 7, 42000 Varaždin, Croatia
[c] Faculty of Textile Technology, University of Zagreb, Pierottijeva 6, 10000 Zagreb, Croatia

ARTICLE INFO

Article history:
Received 6 February 2010
Received in revised form 12 December 2010
Accepted 6 April 2011

Keywords:
2-point and 3-point Radau and Lobatto quadrature formulae
Montgomery identity

ABSTRACT

We present a weighted generalization of Montgomery identity for Riemann–Stieltjes integral and use it to obtain weighted generalization of a recently obtained inequality, as well as weighted 2-point and 3-point quadrature formulae of closed and semi-closed type for functions of bounded variation.

1. Introduction

Dragomir et al. in [1] established the following identity:

Theorem 1. *Let $f : [a, b] \to \mathbb{R}$ be a bounded function on $[a, b]$, and $x_1, x_2, x_3 \in [a, b]$ such that $x_1 \leq x_2 \leq x_3$. Then the following identity holds*

$$\frac{x_1 - a}{b - a} f(a) + \frac{x_3 - x_1}{b - a} f(x_2) + \frac{b - x_3}{b - a} f(b) = \frac{1}{b - a} \int_a^b f(t)\mathrm{d}t + \frac{1}{b - a} \int_a^b S(x_1, x_2, x_3, t)\mathrm{d}f(t) \tag{1.1}$$

where $S(x_1, x_2, x_3, t)$ is defined by

$$S(x_1, x_2, x_3, t) = \begin{cases} t - x_1, & a \leq t \leq x_2, \\ t - x_3 & x_2 < t \leq b. \end{cases}$$

If we take $x_1 = a, x_3 = b$ the identity (1.1) reduces to a *Montgomery identity* for Riemann–Stieltjes integral (see for instance [2])

$$f(x) = \frac{1}{b - a} \int_a^b f(t)\mathrm{d}t + \int_a^b P(x, t)\mathrm{d}f(t) \tag{1.2}$$

* Corresponding author. Tel.: +385 16129965; fax: +385 16129946.
E-mail addresses: andrea.aglic@fer.hr (A. Aglić Aljinović), skovac@gfv.hr (S. Kovač), pecaric@hazu.hr (J. Pečarić).

0895-7177/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mcm.2011.04.006