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Convergence theorems for two finite families of asymptotically nonexpansive mappings☆

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ABSTRACT

The purpose of this paper is to study the strong and weak convergence of a finite step iteration process with errors for two finite families of asymptotically nonexpansive mappings in uniformly convex Banach spaces. The results presented in the paper improve and extend some results in Sitthikul and Saejung (2009) [9].

1. Introduction and preliminaries

Throughout this work, we assume that E is a real Banach space and C is a nonempty subset of E. A mapping $T : C \to C$ is said to be asymptotically nonexpansive if there exists a sequence $\{h_n\} \subset [1, \infty)$ with $\lim_{n\to\infty} h_n = 1$ such that

$$\|T^n x - T^n y\| \leq h_n \|x - y\|, \quad \forall x, y \in C, \ \forall n \geq 1.$$

The class of asymptotically nonexpansive self-mappings was introduced by Goebel and Kirk [1] in 1972 as an important generalization of the class of nonexpansive self-mappings, who proved that if C is a nonempty closed convex subset of a real uniformly convex Banach space and T is an asymptotically nonexpansive self-mapping of C, then T has a fixed point.

Since then, iteration processes for asymptotically nonexpansive mappings in Banach spaces have studied extensively by many authors (see [2–6]). In 2002, Xu and Noor [7] introduced and studied a three-step scheme to approximate fixed points of asymptotically nonexpansive mappings in Banach space. Cho et al. [8] extended the work of Xu and Noor to a three-step iterative scheme with errors in Banach space and proved the weak and strong convergence theorems for asymptotically nonexpansive mappings. Recently, Sitthikul and Saejung [9] introduced a finite-step iteration scheme for a finite family of nonexpansive and asymptotically nonexpansive mappings as follows:

Let C be a nonempty convex subset of a Banach space E. Let $\{S_i\}_{i=1}^N : C \to C$ be N nonexpansive mappings, $\{T_i\}_{i=1}^N : C \to C$ be N asymptotically nonexpansive mappings. Then the sequence $\{x_n\}$ defined by

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