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New efficient numerical methods to describe the heat transfer in a solid medium

E. Ponsoda *, S. Blanes, P. Bader

Instituto de Matemática Multidisciplinar, Building 8G, Universidad Politécnica de Valencia, 46022 Valencia, Spain

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ABSTRACT

The analysis of heat conduction through a solid with heat generation leads to a linear matrix differential equation with separated boundary conditions. We present a symmetric second order exponential integrator for the numerical integration of this problem using the imbedding formulation. An algorithm to implement this explicit method in an efficient way with respect to the computational cost of the scheme is presented. This method can also be used for nonlinear boundary value problems if the quasilinearization technique is considered. Some numerical examples illustrate the performance of this method.

1. Introduction

In this work, we consider the numerical integration of the linear matrix differential equation with separated boundary conditions originated by the spatial heat conduction through a solid with local areas of heat generation. Let us denote by z the direction of the heat flow and by Z the total length of the solid medium. If we consider that the flow in other directions is much smaller, the problem can be approximated in one dimension. Then, we define a control volume of length Δz and cross sectional area A, where we can perform an energy balance in order to derive a conservation equation for thermal energy in terms of temperature. This analysis leads to a boundary value problem (BVP) which describes the temperature along the length of the body in the direction of the flow, see [1]. After rescaling $t = z/Z$, the non-autonomous and non-homogeneous BVP is given by

$$\left.\begin{array}{l} T''(t) + p(t)T'(t) + q(t)T(t) = f(t); \\ K_{11}T(0) + K_{12}T'(0) = \gamma_1, \qquad K_{21}T(1) + K_{22}T'(1) = \gamma_2 \end{array}\right\} \tag{1}$$

where $T(t)$ is the temperature, $f(t)$ is the heat generation, and $p(t)$, $q(t)$ are the advection and convection coefficients, respectively, that can depend on the local position t and its cross section A at this point. The first term in the equation corresponds to the conduction in the direction of flow. With appropriate coefficients and boundary conditions, the system (1) describes also a material process in which a solid body is moving out of a hot region and the heat flow is mainly oriented towards the direction of the motion of the body, like a long slab of steel emerging from a furnace or a metal rod undergoing continuous hardening, for example, see [2] for details.

On the other hand, it is known that many relevant engineering problems can be modelled by a second order nonlinear differential equation, say

$$T'' = f(t, T, T'), \quad 0 \le t \le 1,$$

* Corresponding author.
E-mail addresses: eponsoda@imm.upv.es (E. Ponsoda), serblaza@imm.upv.es (S. Blanes), phiba@posgrado.upv.es (P. Bader).

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