



Numerical solution of random differential models

J.-C. Cortés^{a,*}, L. Jódar^a, L. Villafuerte^b, R. Company^a

^a Instituto Universitario de Matemática Multidisciplinar, Universidad Politécnica de Valencia, Edificio 8G, 2on, P.O. Box 22012 Valencia, Spain

^b CEFyMAP, Universidad Autónoma de Chiapas, México, Department of Mathematics, University of Texas at Arlington, Arlington, TX 76019, USA

ARTICLE INFO

Article history:

Received 20 September 2010

Received in revised form 17 December 2010

Accepted 20 December 2010

Keywords:

Random differential equations

Mean square calculus

Numerical solution

ABSTRACT

This paper deals with the construction of a numerical solution of random initial value problems by means of a random improved Euler method. Conditions for the mean square convergence of the proposed method are established. Finally, an illustrative example is included in which the main statistics properties such as the mean and the variance of the stochastic approximation solution process are given.

© 2011 Elsevier Ltd. All rights reserved.

1. Introduction

Random differential equations are powerful tools to model problems involving rates of changes of quantities representing variables under uncertainties or randomness, [1–3]. Many of these models are based on random differential equations of the form

$$\dot{\mathbf{X}}(t) = F(\mathbf{X}(t), t), \quad t_0 \leq t \leq t_e, \quad \mathbf{X}(t_0) = \mathbf{X}_0, \quad (1.1)$$

where \mathbf{X}_0 is a random vector and both the unknown, $\mathbf{X}(t)$, and the right-hand side, $F(\mathbf{X}(t), t)$, are vector stochastic processes. Reliable numerical solutions for problem (1.1) have been studied recently in [4–6]. In this paper, we present a random improved Euler method and we establish its mean square convergence in the fixed station sense. The proof of its convergence can be straightforwardly adapted to the extension of the random framework of others cases such as the so-called modified Euler and Runge–Kutta schemes [7], taking advantage of the approach presented here; comments are added about this issue. Apart from studying random improved Euler scheme in order to obtain approximations of the solution stochastic process, we are also interested in providing approximations of the average and variance functions of the solution because they reveal important information about the statistical behavior of the solution.

This paper is organized as follows. Section 2 deals with some preliminary definitions, results, notation and examples that clarify the presentation of the paper. Section 3 is addressed to the analysis of the mean square convergence of the numerical schemes presented. An illustrative example is included in the last section.

2. Preliminaries

This section deals with some preliminary notation, results and examples that will clarify the presentation of the main results of this paper. Let (Ω, \mathcal{F}, P) be a probability space. In the following, we are interested in second-order real random variables (2-r.v.'s), $Y : \Omega \rightarrow \mathbb{R}$, having a density probability function, $f_Y(y)$, such that $E[Y^2] = \int_{-\infty}^{\infty} y^2 f_Y(y) dy < +\infty$,

* Corresponding author. Tel.: +34 963879144.

E-mail addresses: jccortes@imm.upv.es, jccortes@mat.upv.es (J.-C. Cortés), [ljodar@imm.upv.es](mailto:ljudar@imm.upv.es) (L. Jódar), laura.villafuerte@unach.mx (L. Villafuerte), rcompany@imm.upv.es (R. Company).