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Exact and analytic-numerical solutions of bidimensional lagging models of heat conduction

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ABSTRACT

Lagging models of heat conduction, such as the Dual-Phase-Lag or the Single-Phase-Lag models, lead to heat conduction equations in the form of partial differential equations with delays or to partial differential equations of hyperbolic type, and have been considered to model microscale heat transfer in engineering problems or bio-heat transfer in medical treatments.

In this work we obtain explicit solutions for bidimensional lagging models of heat conduction, with different types of boundary conditions, in the form of infinite series solutions, allowing the construction of analytic-numerical solutions with bounded errors. Numerical examples, showing differences between models and the influence of parameters, are discussed.

1. Introduction

The classical model for describing heat conduction and other transport phenomena is the diffusion equation, the parabolic partial differential equation

$$T_t(\mathbf{r}, t) = \alpha \Delta T(\mathbf{r}, t), \tag{1}$$

where $\alpha > 0$ is the thermal diffusivity, t is time, $\mathbf{r}$ denotes a point in the space domain, Δ is the Laplacian, T the temperature, and T_t denotes partial differentiation with respect to t. It is based on the Fourier law

$$\mathbf{q}(\mathbf{r}, t) = -k\nabla T(\mathbf{r}, t), \tag{2}$$

where $\mathbf{q}$ is the heat flux vector and $k > 0$ the thermal conductivity, combined with the energy conservation equation

$$-\nabla \cdot \mathbf{q}(\mathbf{r}, t) + Q(\mathbf{r}, t) = C_p T_t(\mathbf{r}, t), \tag{3}$$

where C_p is the volumetric heat capacity and Q the volumetric heat source. In the absence of heat sources, Eq. (1) is obtained, where $\alpha = k/C_p$.

This classical model can be successfully applied to conventional technical problems, as it gives accurate macroscopic descriptions for the long term behavior of systems with large spatial dimensions. However, it implies an infinite speed of propagation, which has no physical meaning, and does not explain phenomena such as thermal "inertia", heat waves and delayed responses to thermal disturbances, which appear in transient responses at the microscale level (see [1,2]).

Non-Fourier models of heat conduction have attracted much attention in recent years. Models incorporating time lags, such as the Single-Phase-Lag (SPL) model [3] or the Dual-Phase-Lag (DPL) model [2,4,5], lead to heat conduction

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