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Accurate matrix exponential computation to solve coupled differential models in engineering☆

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ABSTRACT

The matrix exponential plays a fundamental role in linear systems arising in engineering, mechanics and control theory. This work presents a new scaling-squaring algorithm for matrix exponential computation. It uses forward and backward error analysis with improved bounds for normal and nonnormal matrices. Applied to the Taylor method, it has presented a lower or similar cost compared to the state-of-the-art Padé algorithms with better accuracy results in the majority of test matrices, avoiding Padé's denominator condition problems.

1. Introduction

Many engineering processes are described by systems of linear first-order ordinary differential equations with constant coefficients, whose solutions are given in terms of the matrix exponential, and a large number of methods for its computation have been proposed [1,2]. This work presents a competitive new scaling and squaring algorithm for matrix exponential computation. Throughout this paper $\mathbb{C}^{n\times n}$ denotes the set of complex matrices of size $n \times n$, I denotes the identity matrix for this set, $\rho(A)$ is the spectral radius of matrix A, and $\mathbb{N}$ denotes the set of positive integers. The matrix norm $\|\cdot\|$ denotes any subordinate matrix norm; in particular $\|\cdot\|_1$ is the 1-norm. This paper is organized as follows. Section 2 presents the scaling and squaring error analysis and the developed algorithm, and Section 3 deals with numerical tests and conclusions. Next theorem will be used in next section to bound the norm of matrix power series.

Theorem 1. *Let $h_l(x) = \sum_{k\geq l} b_k x^k$ be a power series with radius of convergence w, and let $\tilde{h}_l(x) = \sum_{k\geq l} |b_k| x^k$. For any matrix $A \in \mathbb{C}^{n\times n}$ with $\rho(A) < w$, if a_k is an upper bound for $\|A^k\|$ ($\|A^k\| \leq a_k$), $p \in \mathbb{N}$, $1 \leq p \leq l$, and $\alpha_p = \max\{(a_k)^{\frac{1}{k}} : k = p, l, l+1, \ldots, l+p-1\}$, then $\|h_l(A)\| \leq \tilde{h}_l(\alpha_p)$. If $p = 2$ and l is odd the same bound holds taking $\alpha_2 = \max\{(a_k)^{\frac{1}{k}} : k = 2, l\}$.*

Proof. For the first part note that

$$\|h_l(A)\| \leq \sum_{j\geq 0} \sum_{i=l}^{l+p-1} |b_{i+jp}| \, \|A^p\|^j \|A^i\| \leq \sum_{j\geq 0} \sum_{i=l}^{l+p-1} |b_{i+jp}| \alpha_p^{i+jp} = \sum_{k\geq l} |b_k| \alpha_p^k = \tilde{h}_l(\alpha_p). \quad (1)$$

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