



## Thermal development of the laminar flow of a Bingham fluid between two plane plates with viscous dissipation

Abdelhamid Boualit<sup>a,\*</sup>, Nouredine Zeraibi<sup>b</sup>, Sabrina Boualit<sup>a</sup>, Meriam Amoura<sup>c</sup>

<sup>a</sup>Unité de recherche appliquée en énergies renouvelables, Zone industrielle Noumirat, Ghardaïa, Algeria

<sup>b</sup>Université de Boumerdes, Faculté des hydrocarbures dépt. Transport et équipement, 35000 Avenue de l'indépendance, Boumerdes, Algeria

<sup>c</sup>Université des Sciences et de la Technologie Houari Boumedienne, Faculté de Physique, Dépt. Energétique, B.P. 32 El-Alia, bab-Ezzouar, 16111 Alger, Algeria

### ARTICLE INFO

#### Article history:

Received 26 November 2008

Received in revised form

7 September 2010

Accepted 8 September 2010

Available online 15 October 2010

#### Keywords:

Bingham

Yield stress

Thermal development

Nusselt

Viscous dissipation

Plane plates

### ABSTRACT

The thermal development of the hydrodynamically developing laminar flow of a viscoplastic fluid (fluid of Bingham) between two plane plates maintained at a constant temperature has been studied numerically. This analysis has shown the effect caused by inertia and the rheological behaviour of the fluid on the velocity, pressure and temperature fields. The effects of Bingham and Peclet numbers on the Nusselt values with the inclusion of viscous dissipation are also discussed.

© 2010 Elsevier Masson SAS. All rights reserved.

### 1. Introduction

The laminar forced convective heat transfer in duct flows has already interested the researchers since several decades, because of its foremost importance in the practical thermal systems development.

The first studies were those carried out by Graetz (1883–1885) where he has considered Newtonian fluids in situation of a developed laminar forced convective heat transfer in circular section pipe, subjected to a parietal heating by an imposed constant temperature. By neglecting the effect of viscous dissipation and the axial conduction of the fluid, Graetz led to a simplified formulation of the equation of energy, which was solved only later by W. Nusselt using a variable separation method.

The analysis of Graetz was the beginning of a long series of experimental, analytical and numerical investigations, which considered the problem under less restrictive conditions, more complex geometries and non-Newtonian fluid behaviour. The whole of these studies forms a common field of research commonly called “The Graetz problem extended”. For the Newtonian fluids, Sellars et al. [1] and Siegel et al. [2] have considered the problem of

\* Corresponding author.

E-mail address: [h.boualit@yahoo.fr](mailto:h.boualit@yahoo.fr) (A. Boualit).

Graetz in the case of a uniform walls heat flux condition, while neglecting the effect of viscous dissipation and the axial conduction of the fluids. The effect of the axial conduction has been considered by Hsu [3], where the problem has been solved analytically by the variables separation method. Ou and Cheng [4] have obtained a solution of the Graetz problem in the case of a wall heat flux condition with the consideration of the viscous dissipation effect.

The study of the influence exerted by viscous dissipation on the thermal field has shown that the effect of this one is very significant on heat transfer at the entrance region, in the case of hydrodynamically and thermally developing flows [5,6].

In developed flows, this effect is also noticed. It was reported that the value of the developed Nusselt number, in the case of constant wall temperature condition, is 9.6 considering the effect of viscous dissipation [7], whereas this value is 3.6568 by neglecting it [8]. Kays and Grawford [9] have shown that the effect of viscous dissipation on the thermal field is significant during the Newtonian fluid flows at high velocity.

For the non-Newtonian fluids, Bird et al. [10] have shown that this effect is also significant during the flows at low velocity, because of the high viscosities and the high velocity gradient which characterize this type of fluids.

In the case of a viscoplastic fluid of Bingham, Wissler and Schechter [11], and Blackwell [12] have studied the development of