



# Double-diffusive natural convective boundary layer flow in a porous medium saturated with a nanofluid over a vertical plate: Prescribed surface heat, solute and nanoparticle fluxes

W.A. Khan<sup>a</sup>, A. Aziz<sup>b,\*</sup>

<sup>a</sup>Department of Engineering Sciences, National University of Sciences and Technology, Karachi 75350, Pakistan

<sup>b</sup>Department of Mechanical Engineering, School of Engineering and Applied Science, Gonzaga University, Spokane, WA 99258, USA

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## ABSTRACT

The Buongiorno model [16] has been used to study the double-diffusive natural convection from a vertical plate to a porous medium saturated with a binary base fluid containing nanoparticles. The model identifies the Brownian motion and thermophoresis as the primary mechanisms for enhanced convection characteristics of the nanofluid. The behavior of the porous medium is described by the Darcy model. The vertical surface has the heat, mass and nanoparticle fluxes each prescribed as a power law function of the distance along the wall. The transport equations are transformed into four nonlinear, coupled similarity equations containing eight dimensionless parameters. These equations are solved numerically to obtain the velocity, temperature, solute concentration and nanoparticle concentration in the respective boundary layers. Results are presented to illustrate the effects of various parameters including the exponent of the power law describing the imposed surface fluxes on the heat and mass transfer characteristics of the flow. These results are supplemented with the data for the reduced Nusselt number and the two reduced Sherwood numbers, one for the solute and the other for the nanoparticles.

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## 1. Introduction

Convective heat transfer in nanofluids is a topic of major contemporary interest in the heat transfer research community. The word “nanofluid” coined by Choi [1] describes a liquid suspension containing ultra-fine particles (diameter less than 50 nm). With the rapid advances in nano manufacturing, many inexpensive combinations of liquid/particle are now available. These include particles of metals such aluminum, copper, gold, iron and titanium or their oxides. The base fluids used are usually water, ethylene glycol, toluene and oil. Experimental studies e.g. [2–7], show that even with the small volumetric fraction of nanoparticles (usually less than 5%), the thermal conductivity of the base liquid can be enhanced by 10–50%. The enhanced thermal conductivity of a nanofluid together with the thermal dispersion of particles and turbulence induced by their motion contributes to a remarkable improvement in the convective heat transfer coefficient. This feature of nanofluids make them attractive for use in applications

such as advanced nuclear systems [8] and cylindrical heat pipes [9]. The literature on the thermal conductivity and viscosity of nanofluids has been reviewed by Trisaksri and Wongwises [10], Wang and Mujumdar [11], Eastman et al. [12], and Kakac and Pramuanjaroenkij [13], among several others. These reviews discuss in detail the preparation of nanofluids, theoretical and experimental investigations of thermal conductivity and viscosity of nanofluids, and the work done on convective transport in nanofluids. A benchmark study of thermal conductivity of nanofluids has been published by Buongiorno et al. [14]. This study analyzed the experimental thermal conductivity data gathered by 30 organizations worldwide and found most of them to be consistent within 10%. The study concludes that the thermal conductivity of a nanofluid increases with the particle concentration and aspect ratio in conformity with the classical Maxwell theory which predicts that the effective thermal conductivity ratio  $k/k_f$  is a function of particle volume fraction  $\phi$  and the thermal conductivity ratio  $k_p/k_f$ . This functional dependence is valid for  $\phi \ll 1$  and  $k_p/k_f < 10$ .

Several ideas have been proposed to explain the enhanced heat transfer characteristics of nanofluids. For example, Pak and Cho [4] attributed the increased heat transfer coefficients observed in nanofluids to the dispersion of suspended particles. Xuan and Li [5] suggested that the heat transfer enhancement was the result of the

\* Corresponding author. Tel.: +1 509 313 3540; fax: +1 509 313 5871.

E-mail addresses: [wkhan\\_2000@yahoo.com](mailto:wkhan_2000@yahoo.com) (W.A. Khan), [aziz@gonzaga.edu](mailto:aziz@gonzaga.edu) (A. Aziz).