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# Approximation of artificial satellites' preliminary orbits: The efficiency challenge☆

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ABSTRACT

In this paper the problem of the determination of the preliminary orbit of a celestial body is studied. We compare the results obtained by the classical Gauss's method with those obtained by some higher-order iterative methods for solving nonlinear equations. The original problem of the determination of the preliminary orbits was posed by means of a nonlinear equation. We modify this equation in order to obtain a nonlinear system which describes the mentioned problem and we derive a new efficient iterative method for solving it. We also propose a new definition of optimal order of convergence for iterative methods for solving nonlinear systems.



## 1. Introduction

Finding the simple roots of a nonlinear equation $f(x) = 0$ or a nonlinear system $F(x) = 0$ are common and important problems in science and engineering. In recent years, many modified iterative methods have been developed to improve the local order of convergence of some classical methods such as the Newton, Potra–Pták, Chebyshev, Halley and Ostrowski's methods.

As the order of an iterative method increases, so does the number of functional evaluations per step. The efficiency index (see [1]), gives a measure of the balance between those quantities, according to the formula $p^{1/d}$, where $p$ is the order of the method and $d$ the number of functional evaluations per step. Kung and Traub [2] conjecture that the order of convergence of any multipoint method without memory cannot exceed the bound $2^{d-1}$, (called the *optimal order*). Ostrowski's method [1], Jarrat's method [3] and King's method [4] are some of optimal fourth order methods.

More recently, some optimal eight order methods have been proposed, see for example [5,6]. In [7] the authors derive an optimal eighth order method, denoted MOP8, starting from the well known third order Potra–Pták's method, by composing it with the modified Newton's iterations and approximating several function evaluations in order to improve the efficiency.

In the multidimensional case, it is also important to take into account the number of operations performed, since for each iteration a number of linear systems must be solved. We recall that the number of products/quotients that we need for solving $m$ linear systems with the same matrix of size $n \times n$, by using $LU$ factorization, is $\frac{1}{3}n^3 + mn^2 - \frac{1}{3}n$, $n = 2, 3, \ldots$. For this reason, in [8] the authors defined the *Computational Efficiency Index* as $\mathrm{CI} = p^{1/(d+op)}$, where $op$ is the number of products/quotients per iteration. For example, the computational efficiency index of Newton's method is $\mathrm{CI}_N = 2^{\frac{1}{(1/3)n^3+2n^2+(2/3)n}}$.

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