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Balancing consistency and expert judgment in AHP

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ABSTRACT

The various mechanisms that represent the know-how of decision-makers are exposed to a common weakness, namely, a lack of consistency. To overcome this weakness within AHP (analytic hierarchy process), we propose a framework that enables balancing consistency and expert judgment. We specifically focus on a linearization process for streamlining the trade-off between expert reliability and synthetic consistency. An algorithm is developed that can be readily integrated in a suitable DSS (decision support system). This algorithm follows an iterative feedback process that achieves an acceptable level of consistency while complying to some degree with expert preferences. Finally, an application of the framework to a water management decision-making problem is presented.

1. Introduction

One of the best established and most modern models of decision-making is AHP (analytic hierarchy process) [1–3]. In AHP, the input format for decision-makers to express their preferences derives from pair-wise comparisons among various elements. Comparisons can be determined by using, for instance [4], a scale of integers 1–9 to represent opinions ranging from 'equal importance' to 'extreme importance' [5] (intermediate decimal values are sometimes useful). Homogeneous and reciprocal judgment yields an $n \times n$ matrix A with $a_{ii} = 1$ and $a_{ij} = 1/a_{ji},\ i, j = 1, \dots, n$. This last property is called *reciprocity* and A is said to be a *reciprocal matrix*. The aim is to assign to each of n elements, E_i, priority values $w_i,\ i = 1, \dots, n$, that reflect the emitted judgments. If judgments are consistent, the relations between the judgments a_{ij} and the values w_i turn out to be $a_{ij} = w_i/w_j,\ i, j = 1, \dots, n$, and it is said that A is a *consistent matrix*. This is equivalent to $a_{ij}a_{jk} = a_{ik}$ for $i, j,\ k = 1, \dots, n$ [6]. As stated by [7,2], the leading eigenvalue and the principal (Perron) eigenvector of a comparison matrix provides information to deal with complex decisions, the normalized Perron eigenvector giving the sought priority vector. In the general case, however, A is not consistent. The hypothesis that the estimates of these values are small perturbations of the 'right' values guarantees a small perturbation of the eigenvalues (see, e.g., [8]). Now, the problem to solve is the eigenvalue problem $A\mathbf{w} = \lambda_{\max}\mathbf{w}$, where $\lambda_{\max}$ is the unique largest eigenvalue of A that gives the Perron eigenvector as an estimate of the *priority vector*.

As a measurement of inconsistency, Saaty [5] proposed using the consistency index $CI = (\lambda_{\max} - n)/(n - 1)$ and the consistency ratio $CR = CI/RI$, where RI is the so-called average consistency index [5]. If $CR < 0.1$, the estimate is accepted; otherwise, a new comparison matrix is solicited until $CR < 0.1$. To overcome inconsistency in AHP while still taking into account expert know-how, the authors propose a model to balance the latter with the former. Our model incorporates an extended version of the linearization procedure described in [9], and integrates it along with AHP to produce optimal comparison matrices.

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